

Tests for Poisson process

- **Exponential distribution of interarrival times.**

First we should compare $\hat{m}$ and $\hat{\sigma}$.

In the case of interarrival times in *Bank data* we get

$$\hat{m}_a = 0.939; \hat{\sigma}_a = 0.859; \hat{c}_a = 0.915.$$

Then we can use a goodness-of-fit test, χ^2 for example. Figure 5 and Table 2 below present the results for interarrival times. Distribution $\chi^2(3)$ is used. (Why are there 3 degrees of freedom?) The hypothesis H_0 is not rejected.

Figure 5

Bank data. Histogram of interarrival times.

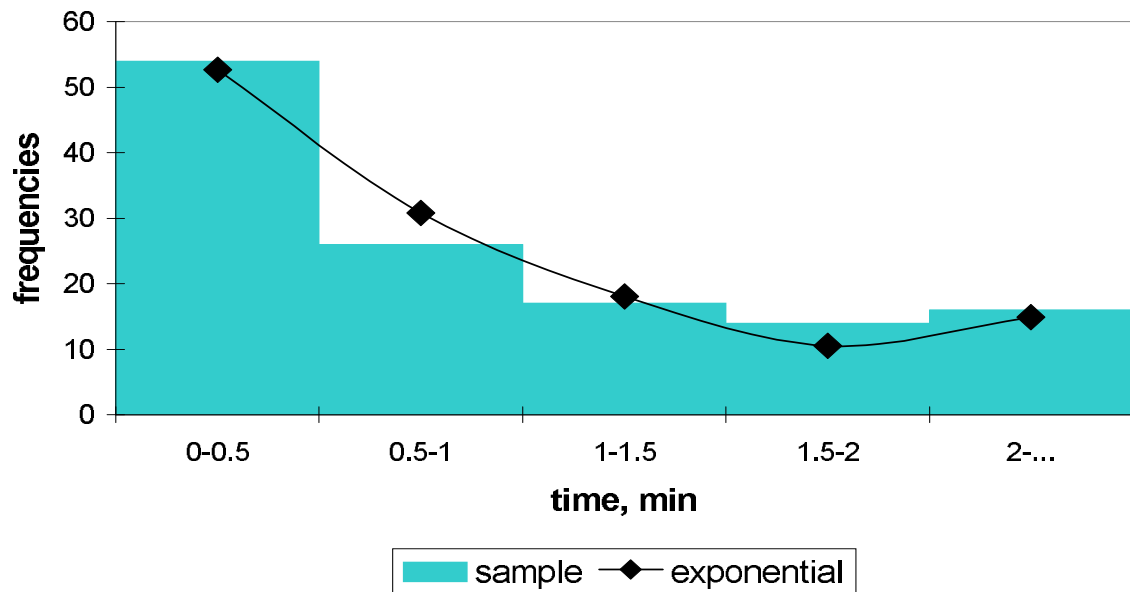


Table 2

Interval	Theoretical	Frequency	
0-0.5	52.69	54	0.03
0.5-1	30.83	26	0.76
1-1.5	18.04	17	0.06
1.5-2	10.55	14	1.13
2-...	14.88	16	0.08
Chi-square			2.06
p-value			0.56

Remark. If the underlying distribution is exponential, there is a formula for the (asymptotically) optimal bin width (with respect to the best L^2 -approximation of the density). Specifically, if N is the number of observation and m is the distribution average

$$d_{opt} \approx \left(\frac{12}{N}\right)^{1/3} \cdot m.$$

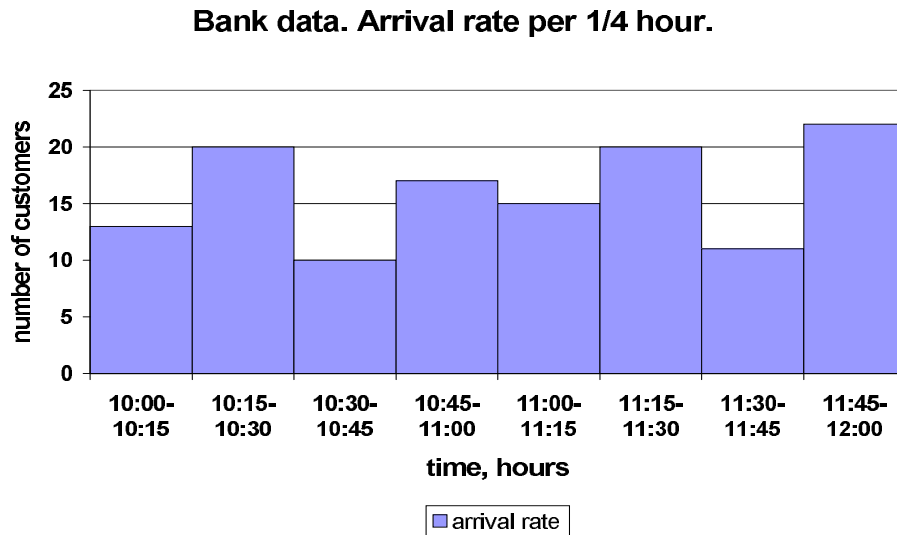
In our interarrival case

$$d_{opt} = \left(\frac{12}{127}\right)^{1/3} \cdot 0.939 = 0.43,$$

close to the 0.5 bin width of the histogram.

- **Time-homogeneity of Poisson process:**

We divide the time interval in question to equal subintervals (1/4 hour in our case) and plot the number of arrivals during those subintervals. We check if any regular pattern is exposed.



Now we perform a χ^2 test:

Table 3

Interval	Theoretical	Frequency	
10-10.25	16	13	0.56
10.25-10.5	16	20	1.00
10.5-10.75	16	10	2.25
10.75-11	16	17	0.06
11-11.25	16	15	0.06
11.25-11.5	16	20	1.00
11.5-11.75	16	11	1.56
11.75-12	16	22	2.25
Chi-square	8.75		
p-value	0.27		

Then we use the following property of the Poisson process: given a fixed number of arrivals in an interval, the numbers of arrivals in non-overlapping subintervals have jointly a **Multinomial distribution**. If the Poisson process is homogeneous and the subintervals are of equal length, then the multinomial probabilities are equal ($1/8$ in our case). The chi-square test with 6 degrees of freedom is performed below for the arrival data in order to compare empirical arrival frequencies with the multinomial distribution. H_0 is not rejected.

Summarizing all tests that we have done with the interarrival times from *Bank Data*, we see no support for rejecting the Poisson hypothesis.