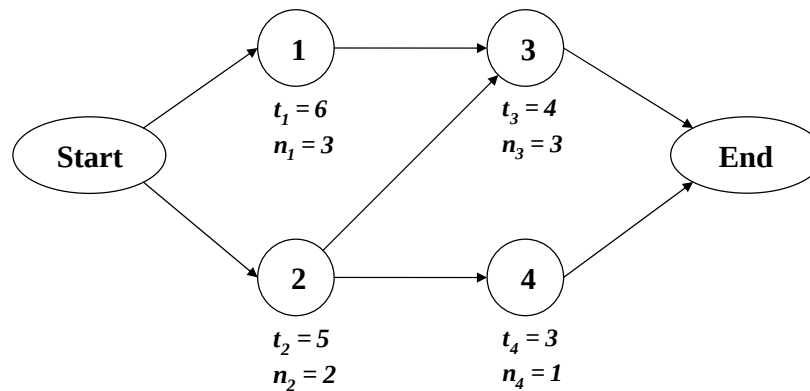


Part 3 - Capacity of a Fork-Join Network:

Tell-On company is a supplier of customized telephone switching equipment. The company buys standard electronic modules from outside suppliers, then modifies and assembles them in accordance with customer specifications. The assembly of a Tell-On system may be thought of as a project with four constituent tasks, each performed by a different team of specialists. The precedence constraints among those tasks are expressed by the **precedence** diagram below.

Note: For a project to be completed (reach the End state), all of its tasks must be completed. Furthermore, before a task can start, all of its predecessor tasks must be completed. e.g. task 2 must be completed before tasks 3 or 4 may start. (Precedence diagrams are commonly used in PERT/CPM).



- (a) Suppose first that each task i takes exactly t_i days (numerical values are shown on the diagram), there is one team devoted to each task, each team begins its task as soon as possible, and there is just one customer order to be dealt with. How long will it take to fill the order? Explain briefly your answer.
- (b) Consider a more interesting dynamic scenario where new customer orders actually arrive according to a Poisson process at an average rate of 0.28 per day. Assume that the system operates in steady-state. Since all tasks must be performed for all projects, the arrival rates of tasks to teams 1-4 are all equal to 0.28 tasks per day. Also, the times required for the tasks involved in completing such orders are independent exponentially distributed random variables, and t_i is now interpreted as the *expected* time to complete task i . Finally, there are n_i different specialist teams dedicated to task i (these values are shown on the network diagram). Teams execute customer orders on a first-in-first-out basis; each task i associated with any given order is begun as soon as its predecessor tasks have been completed and there is a qualified specialist team available. The following three questions pertain to this dynamic scenario:
 - (b.1) The average time to complete a customer order (that is, the average response time seen by customers) will be longer than your answer in part (a), perhaps much longer. Explain why. (Do not carry out any calculations; a qualitative answer suffices.)

- (b.2) What is the utilization for each of the four types of specialist teams? That is, for each type of team, what is the long-run fraction of available team hours that will actually be spent working rather than idle?
- (b.3) Suppose that you could invest in a process improvement that would reduce the average task time for one of the four tasks, leaving everything else unchanged. For which task(s) is such a reduction likely to have the biggest impact on the average response time seen by customers? Why? (Again, provide a qualitative answer.)

Solution:

- (a) The critical path goes through tasks 1 and 3. It will take $6+4=10$ days to fill the order.

- (b.1) The DS-PERT average time to complete a customer order is longer than in part (a) for two reasons:

First, the projects may be delayed in queues for service resources;

second, even if there are no queues for resources, the average completion time of stochastic PERT (exponential task times) is larger than the completion time of deterministic PERT.

- (b.2) Utilization:

$$U_1 = \frac{0.28 \cdot 6}{3} = 0.56; \quad U_2 = \frac{0.28 \cdot 5}{2} = 0.7;$$

$$U_3 = \frac{0.28 \cdot 4}{3} = 0.37; \quad U_4 = \frac{0.28 \cdot 3}{1} = 0.84 .$$

- (b.3) The biggest impact on performance is likely to be reached if we decrease the average task time for **task 4**.

First, its utilization is the highest one.

Second, only one team (server) works there.

(note that in order to get an exact solution to such questions one need to use simulation or analytical analysis.)