Empirical Models

- Case 1. Printing Shop Example (p.2-10)
- Case 2. Face-to-Face Service Operation Example (p. 11-20)
- Using Pivot Table Tool in EXCEL (p. 21-27)

Case 1. Printing Shop Example

- Jobs at a printing shop from 9AM to 12:20PM

Customer	Arrival time	Depart queue	Depart system
1	9:03	9:03	9:23
2	9:10	9:23	9:55
3	9:40	9:55	10:35
4	10:05	10:35	10:50
5	10:15	10:50	11:20
6	10:40	11:20	11:35
7	11:50	11:50	12:20

- **Q1** Calculate the average time spent in queue and in the system
- **Q2** Plot the cumulative arrivals, cumulative departures from queue, and cumulative departures from the system
- **Q3** For 9:00 - 12:20, determine the average number of customers in queue and in the system.
- **Q4** From your answers to Q1 and Q3, verify Little's formula for the customers in queue and in the system in the time interval 9:00-12:20
- **Q5** What would happen to the law, if you consider only the time interval 9:00-12:00?
- **Q6** Determine the throughput and utilization over 9:00 - 12:20

Case 1 - Q1

- **Q1**: Average time spent in queue and in the system
- Define
  - $A^{-1}(n)$: time of the n[th] arrival
  - $D_q^{-1}(n)$: time of the n[th] departure from queue
  - $D_{sys}^{-1}(n)$: time of the n[th] departure from system
  - $W_q(n)$: time in queue, for the n[th] customer to arrive. $W_q(n) = D_q^{-1}(n) - A^{-1}(n)$
  - $W_{sys}(n)$: time in system, for the n[th] customer to arrive. $W_{sys}(n) = D_{sys}^{-1}(n) - A^{-1}(n)$

Customer	Arrival time	Depart queue	Depart system
1	9:03	9:03	9:23
2	9:10	9:23	9:55
3	9:40	9:55	10:35
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5	10:15	10:50	11:20
6	10:40	11:20	11:35
7	11:50	11:50	12:20

n	$W_q(n)$	$W_{sys}(n)$
1	0	20
2	13	45
3	15	55
4	30	45
5	35	64
6	40	55
7	0	30
Avg	19 min	45 min

$\overline{W}_q$ $\overline{W}_{sys}$

- $\overline{W}_q$: average time spent in queue
- $\overline{W}_{sys}$: average time spent in system (queue + service)
- $\overline{W}_{service}$: average time spent in service ($\overline{W}_{sys} - \overline{W}_q = 26$ min)

Case 1 – Q2

- **Q2**: Plot the cumulative arrivals, cumulative departures from queue, and cumulative departures from the system.

Define

- A(t): cumulative arrivals from time 0 to time t
- $D_q(t)$: cumulative departures from the queue from time 0 to time t
- $D_{sys}(t)$: cumulative departures from the system from time 0 to time t

min	A(t)	D_q(t)	D_{sys}(t)
0	0	0	0
0.5	0	0	0
1	0	0	0
1.5	0	0	0
2	0	0	0
2.5	0	0	0
3	1	1	0
3.5	1	1	0
		:	
9	1	1	0
9.5	1	1	0
10	2	1	0
10.5	2	1	0
		:	
21	2	1	0
21.5	2	1	0
22	2	2	1
22.5	2	2	1

Case 1 – Q2 continued

- **Q2**: Plot the cumulative arrivals, cumulative departures from queue, and cumulative departures from the system.

$$A(t) \geq D_q(t) \geq D_{sys}(t).$$

Customer	Arrival time	Depart queue	Depart system
1	9:03	9:03	9:23
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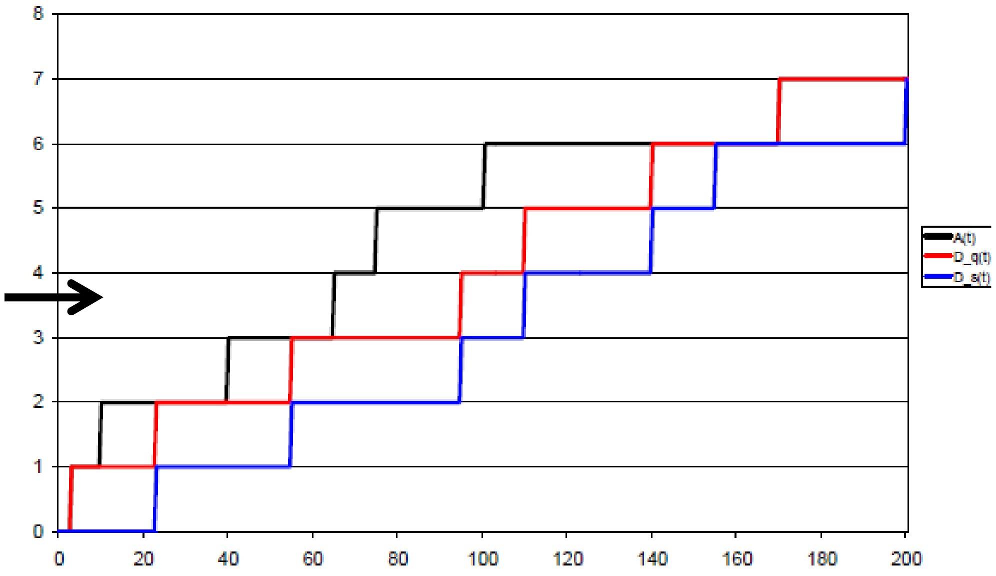
3	9:40	9:55	10:35
4	10:05	10:35	10:50
5	10:15	10:50	11:20
6	10:40	11:20	11:35
7	11:50	11:50	12:20

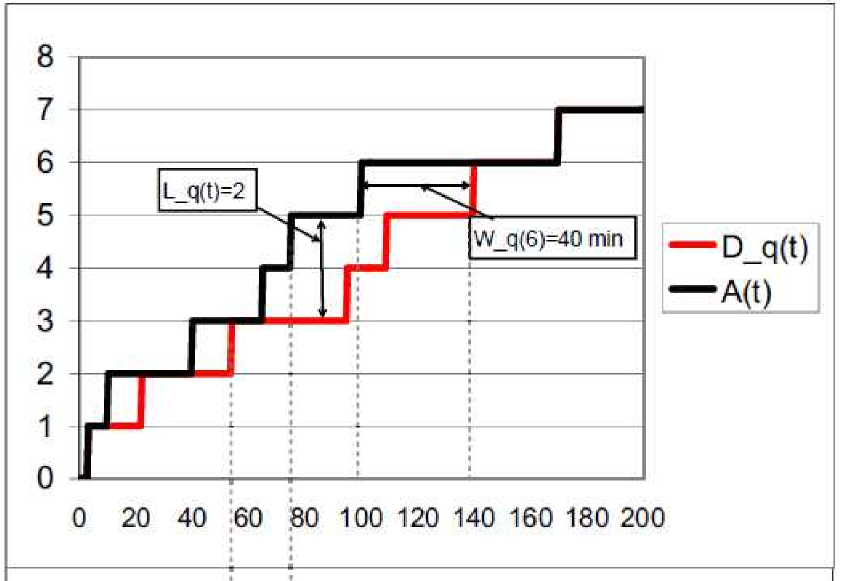


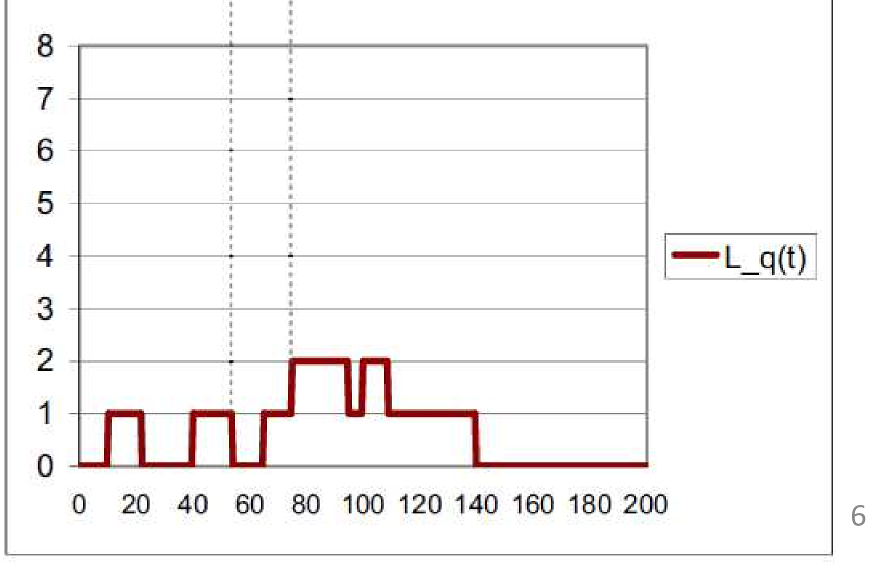
Case 1 – Q3

- **Q3** For 9:00 - 12:20, determine the average number of customers in queue and in the system.

- Define
  - $L_q(t)$: number of customers in the queue at time t. $L_q(t) = \mathrm{A(t)} - \mathrm{D_q(t)}$.
  - $L_{sys}(t)$: number of customers in the system at time t. $L_{sys}(t) = \mathrm{A(t)} - \mathrm{D_{sys}(t)}$.

  - $\bar{L}_\mathrm{q} = \frac{\int_a^b L_q(t)dt}{b-a}$: average queue length
    - The numerator is the area below the graph of $L_q(t)$
  - $\bar{L}_{sys} = \frac{\int_a^b L_{sys}(t)dt}{b-a}$: average number of customers in the system





Case 1 – Q3 continued

- Then,

- $\bar{L}_{\mathrm{q}} = \frac{\int_0^{200} L_q(t)dt}{200} = \frac{(13+15+10+5+30)\times 1+(20+10)\times 2}{200} = 0.6651\ cust$

- $\bar{L}_{\mathrm{sys}} = \frac{\int_0^{200} L_{sys}(t)dt}{200}$

$$= \frac{(7+17+10+15+30)\times 1+(13+15+10+5+30)\times 2+(20+10)\times 3}{200} = 1.575\ cust$$

Case 1 – Q4

- **Q4**: Verify Little's formula for the customers in the time interval 9:00-12:20

- We have
  - $\lambda = \frac{7}{300} = 0.035 \frac{\text{cust}}{\text{min}}$
  - $W_q = 19\ min$ (from Q1)
  - $W_{sys} = 45\ min$ (from Q1)

$\rightarrow \hat{L}_q^{[9:00,12:20]} = 0.035 \frac{cust}{min} \times 19\ mins = \mathbf{0.665\ cust}$

- $\rightarrow$ same as the answer to Q3

$\rightarrow \hat{L}_{sys}^{[9:00,12:20]} = 0.035 \frac{cust}{min} \times 45\ mins = \mathbf{1.575\ cust}$

- $\rightarrow$ same as the answer to Q3

Case 1 – Q5

- **Q5**: Does Little's formula hold for time interval 9:00-12:00?
- We have
  - $\lambda = \frac{7}{300} = 0.035\,\frac{\text{cust}}{\text{min}}$
  - $\overline{\boldsymbol{W}}_{\text{q}} = \frac{0+13+15+30+35+40+0}{7} = 19 \text{ minutes}$
  - $\overline{\boldsymbol{W}}_{\text{sys}} = \frac{20+45+55+45+64+55+\mathbf{10}}{7} = 42.14 \text{ minutes}$
  - $\overline{\boldsymbol{L}}_{\text{q}} = \frac{\int_0^{180} L_q(t)dt}{180} = \frac{(13+15+10+5+\mathbf{10})\times 1+(20+10)\times 2}{180} = 0.6279\ cust$
  - $\overline{\boldsymbol{L}}_{\text{sys}} = \frac{\int_0^{180} L_{sys}(t)dt}{180}$

$$= \frac{(7+17+10+15+\mathbf{10})\cdot 1 + (13+15+10+5+30)\cdot 2 + (20+10)\cdot 3}{180} = 1.6389\ cust$$

$\rightarrow \widehat{\boldsymbol{L}}_{\boldsymbol{q}}^{[9:00,12:00]} = 0.035\,\frac{cust}{min} \times 19\ mins = \mathbf{0.665}\ cust$

Little's law doesn't hold

$\rightarrow \widehat{\boldsymbol{L}}_{\boldsymbol{sys}}^{[9:00,12:00]} = 0.035\,\frac{cust}{min} \times 42.14\ mins = \mathbf{1.475}\ cust$

Case 1 – Q6

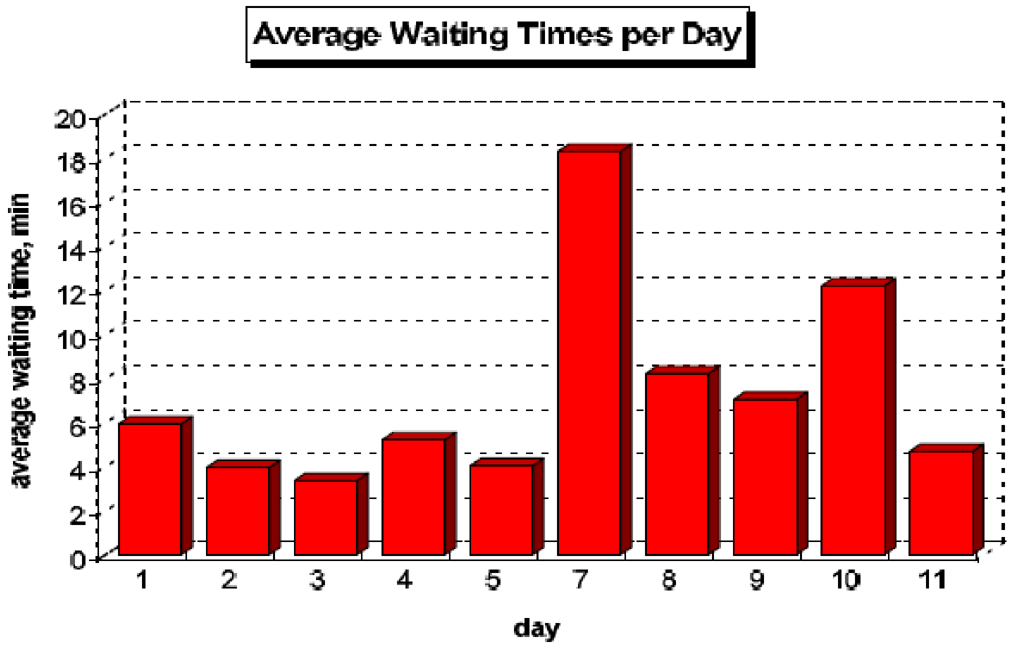
- **Q6**: Throughput and utilization over 9:00 - 12:20

- Throughput rate
  - $\frac{7\ arrivals}{200\ minutes} = 0.035\ per\ minute$

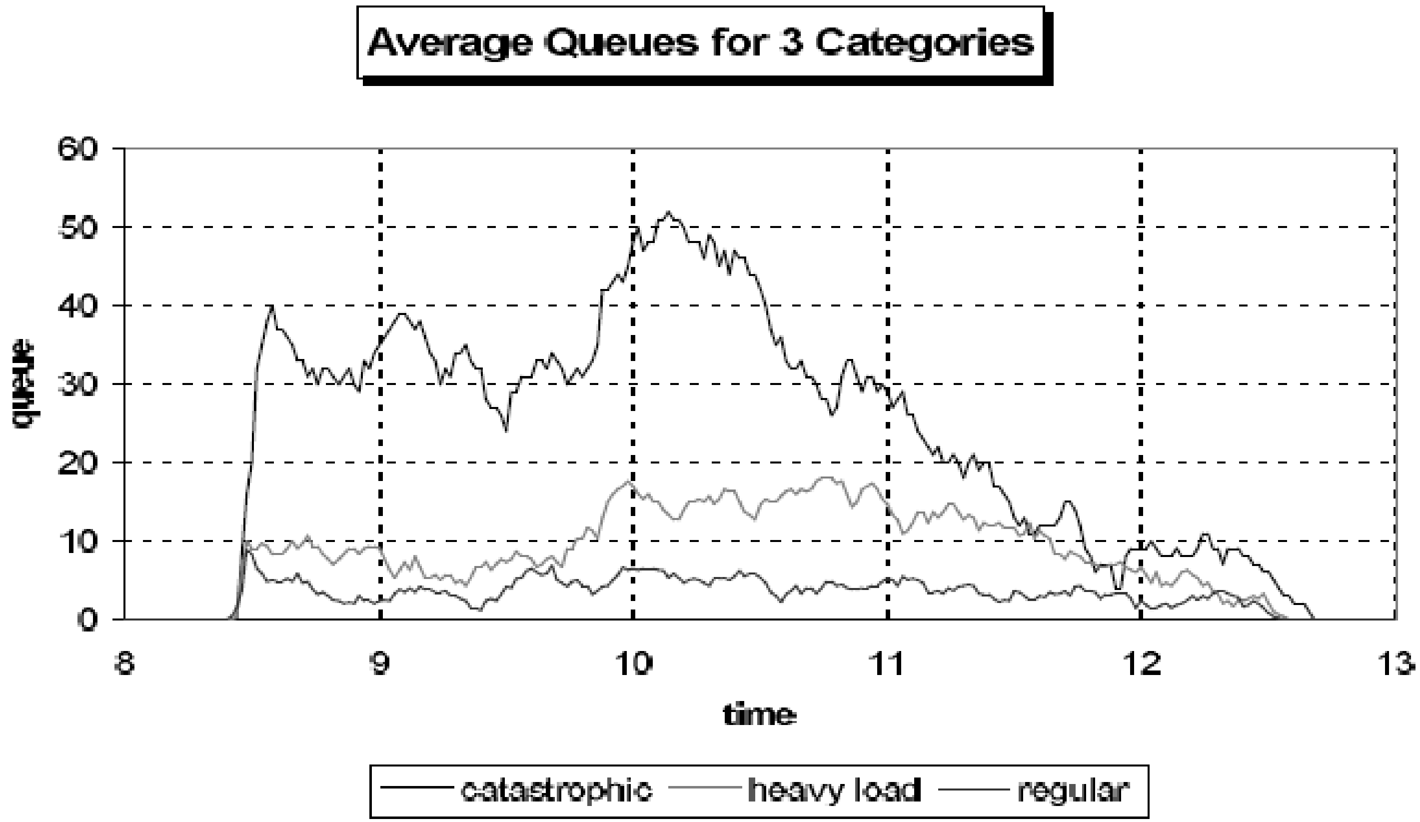
- Utilization
  - $\rho = \frac{busy\ time}{200} = \frac{200-3-15}{200} = 0.91\ (91\%)$

Case 2. Face-to-Face Service of a Bank

- Data from **12 days** of work in a Face-to-Face service of a bank Several servers (maximum 5) work simultaneously at a single station
- Working days were divided into three categories (Data from Fridays (day 6, 12) is not considered because they are different from the others)
    - Catastrophic day: day 7
    - Heavily loaded days: days 8, 9 and 10
    - Regular days: days 1, 2, 3, 4, 5 and 11



Case 2. Queues (1)



- Common features and queue pattern for each day?

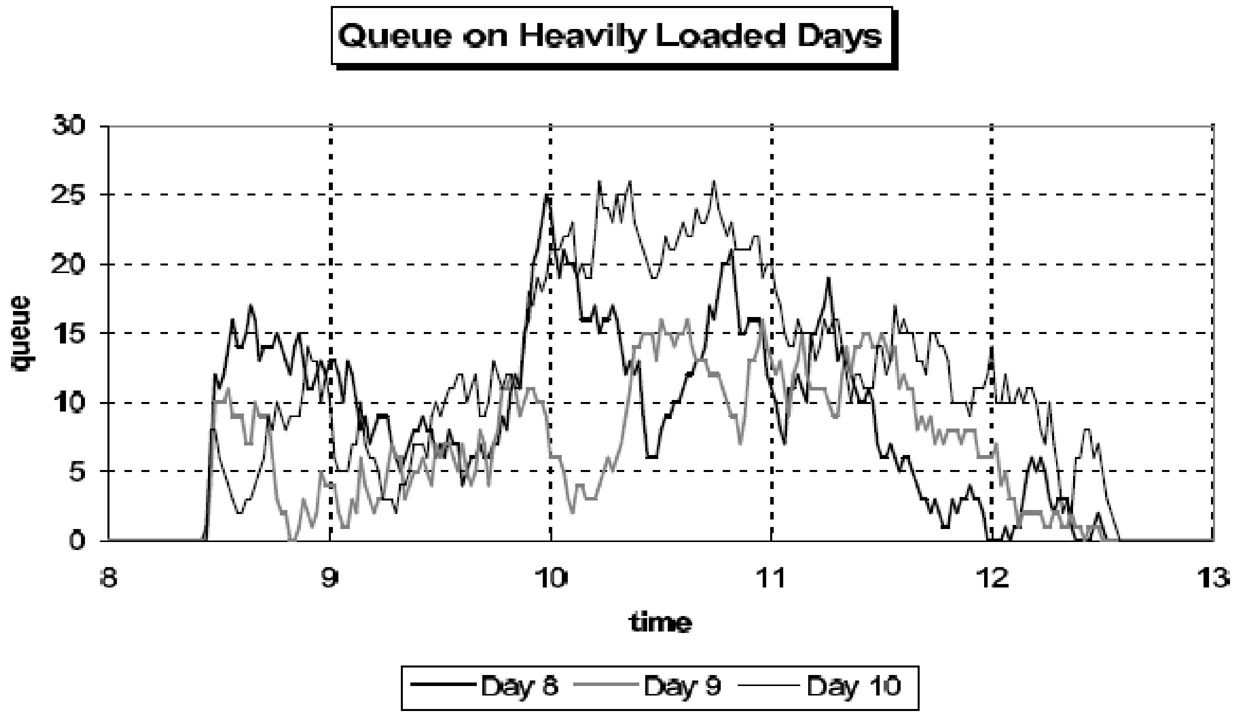
Case 2. Queues (2)

- Common features for different categories.
  - Sharp growth of the queues at the start of the day
    - At 8:30AM, the bank opens and there are people waiting at the door
  - Queue decrease before 9:30
  - Queue growth after 10:00
  - Gradual decrease to zero at the end of the day

- Average queue pattern for every category:
  - Catastrophic day (1 day)
    - Sharp increase when the day starts (40 customers shortly after 8:30)
    - Until 9:30, the queue goes down to 25 customers and then grows rapidly again
    - Approximately at 10:10, we get the record queue for all days of more then 50 customers
    - Then the queue gradually decreases to zero at the end of the day

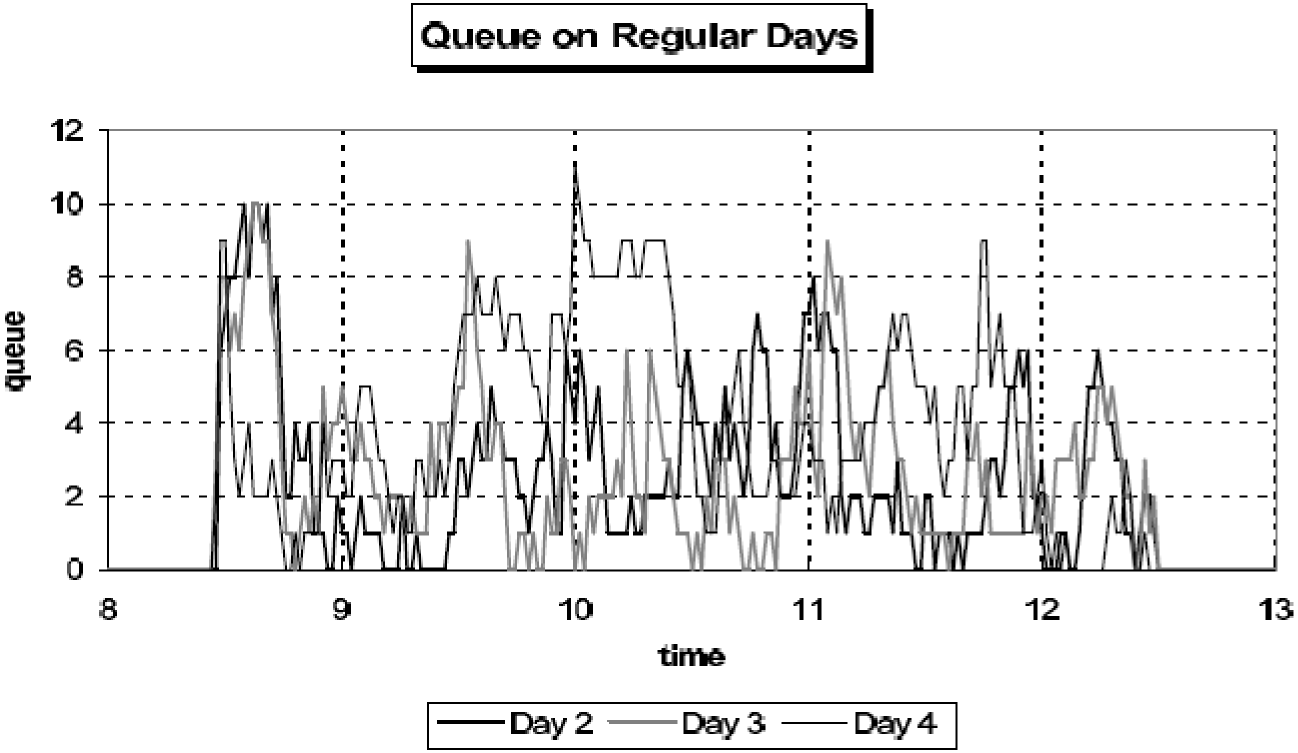
Case 2. Queues (3)

- Heavily loaded days (Average of 3 days)
  - A sharp increase to 10 customers at the beginning of the day, and then oscillates between 5 and 10 customers until 9:45
  - After 11:00 the queue slowly decreases to zero
  - Sample path of each day:

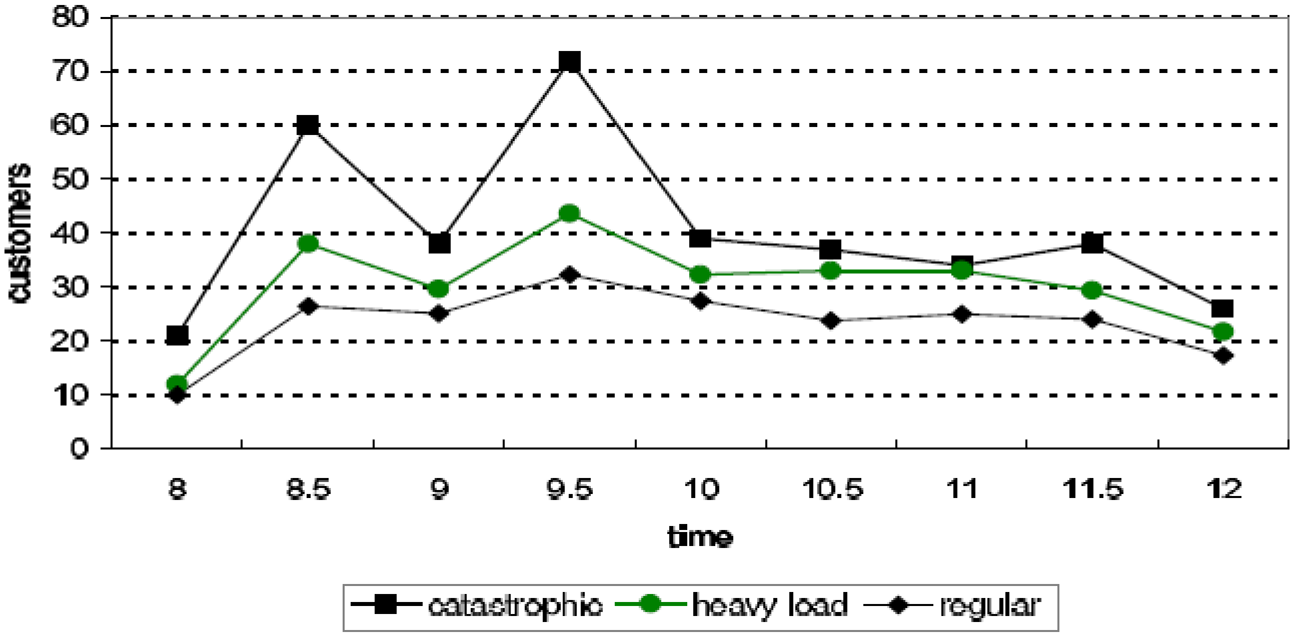


Case 2. Queues (4)

- Regular days (Average of 6 days)
  - jumps to approximately 10 at the beginning
  - decreases close to zero before 9:30
  - over almost the whole day, oscillates in “steady state” near 5
  - Sample path of each day:

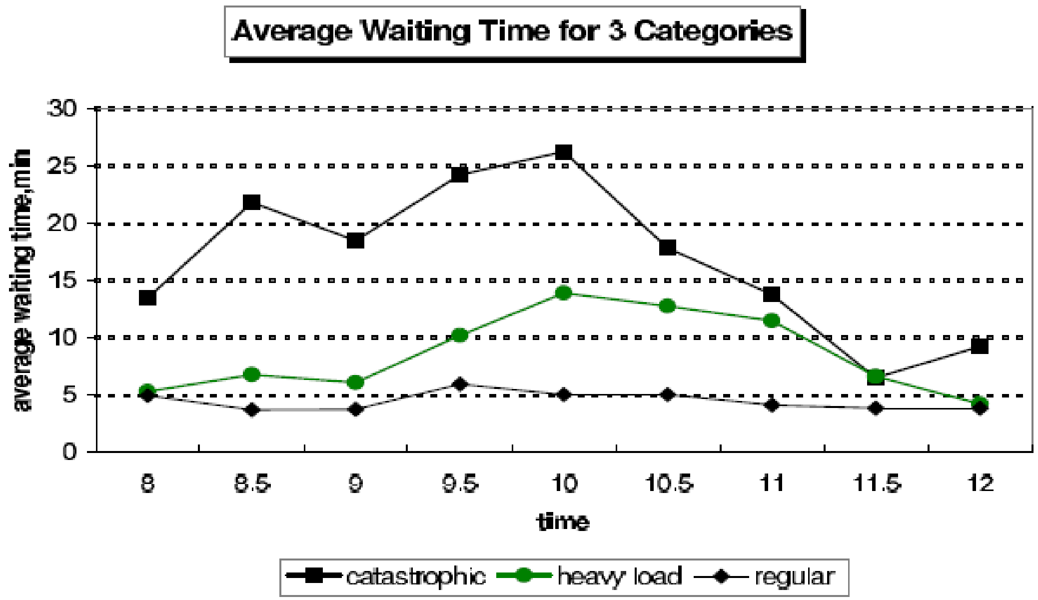


Case 2. Arrival Rates



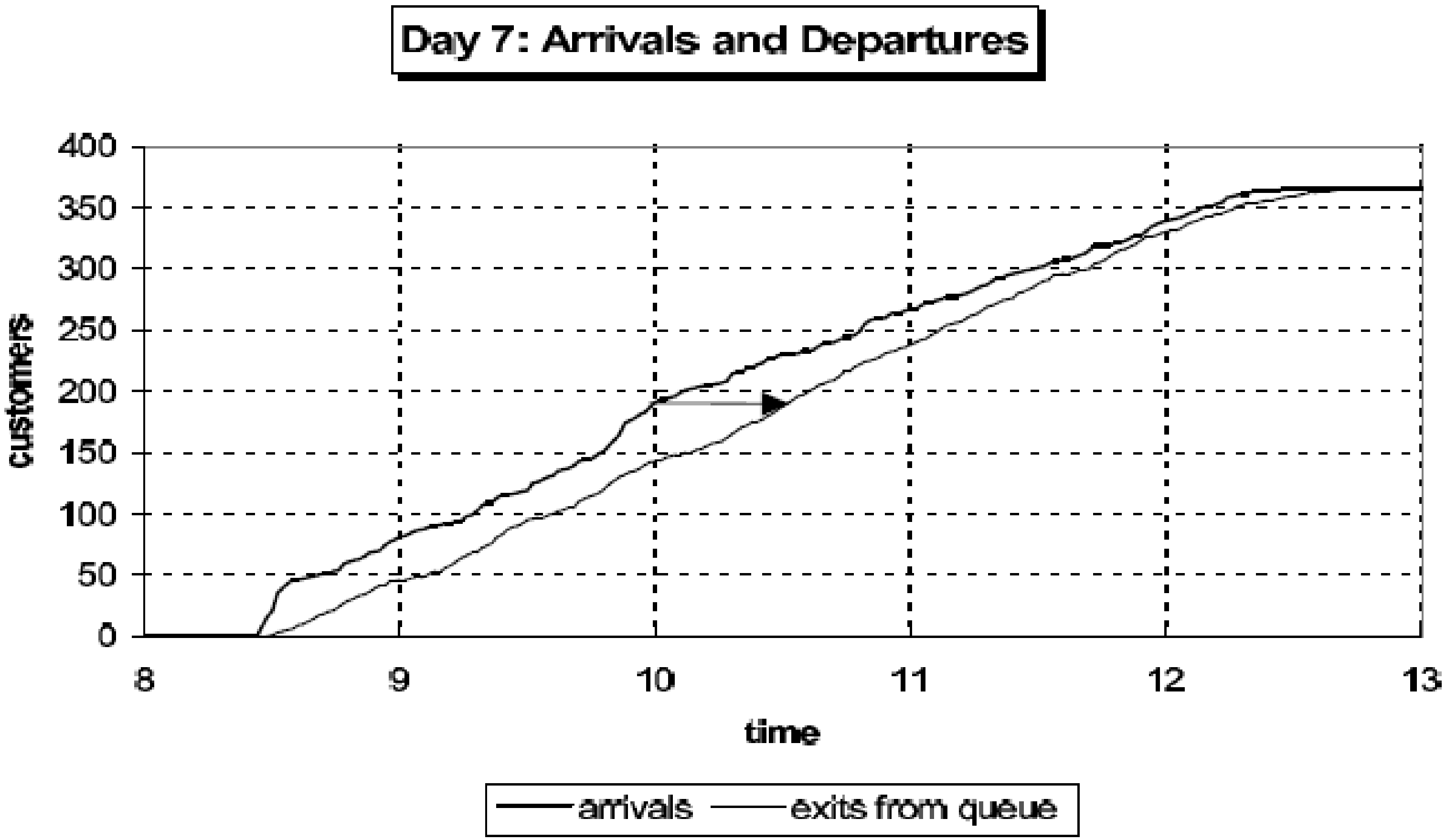
- We have maximum arrival rate at the same time interval (9:30–10:00) for all categories
- The maxima are “steeper” for the categories with a large arrival rate.
- **Catastrophic day** has two sharp peaks of arrivals: 60 customer between 8:30 and 9:00 and more then 70 customers between 9:30 and 10:00. Busy morning.
- **Heavily loaded days** has two maximums at 8:30-9:00 and 9:30-10:00 (approximately 40 customers). For other time intervals, except for the first and the last, the arrival rate is nearly constant (30 customers).
- For **regular days**, the variability of the average arrival rate is small.

Case 2. Waiting Times

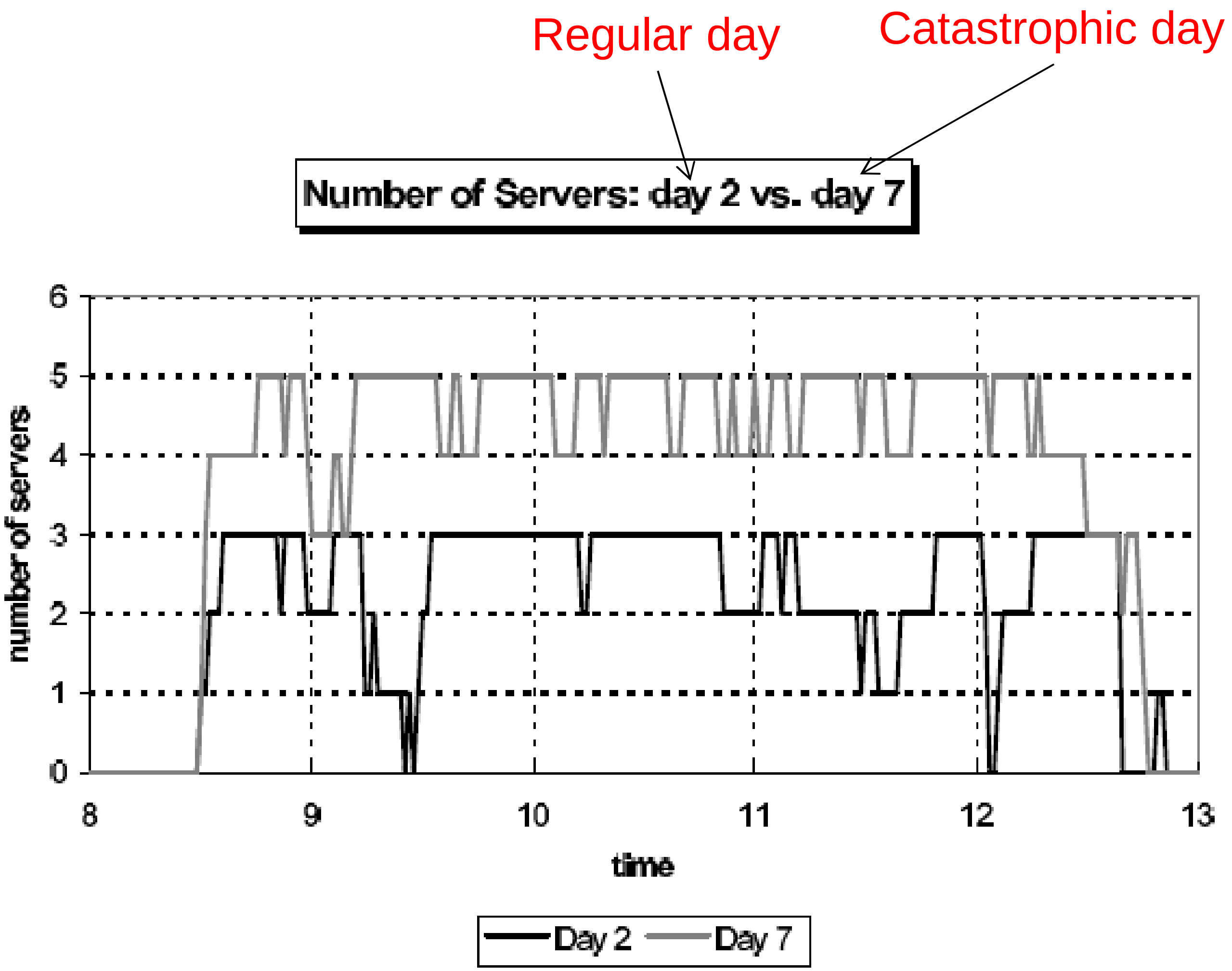


- For catastrophic and heavily loaded days, maximum value at [10:00, 10:30]
    - For a heavily loaded queueing system, the peak of the waiting time usually comes *after* the peak of the arrival rate (Slide 16)
- The patterns of the waiting times follow the **patterns of queues** (slide 12)
- For **Catastrophic day,** rapid decrease after 10:30
- For **heavily loaded days**, the average waiting time is relatively small until 9:30 (5-6 minutes). In the time interval 9:30-11:30 it is over 10 minutes with maximum between 10 and 10:30 (approximately 14 minutes). After 11:30 the waiting times are small.
- For **regular days**, the average waiting time does not change considerably (oscillates around 5 minutes) with time. The maximum takes place in the interval 9:30-10:00 .

Case 2. Waiting Times - Estimate

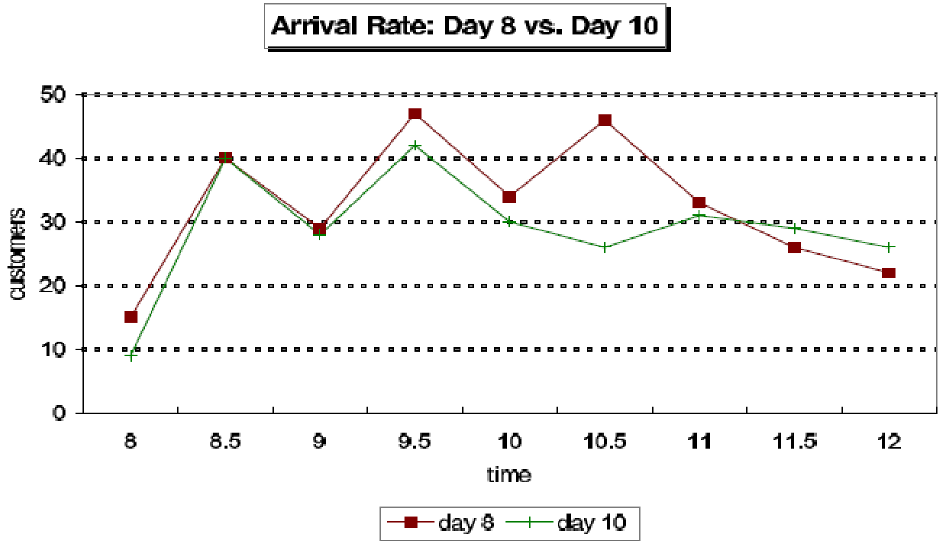


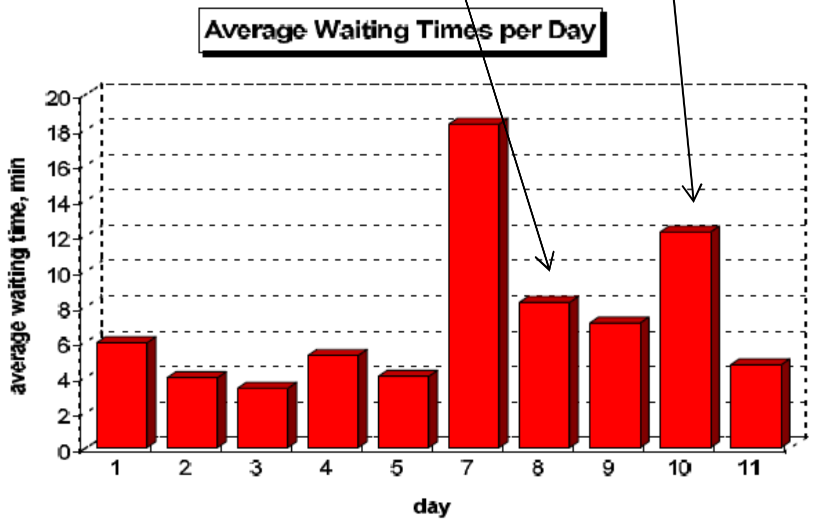
Case 2. Staffing (1)

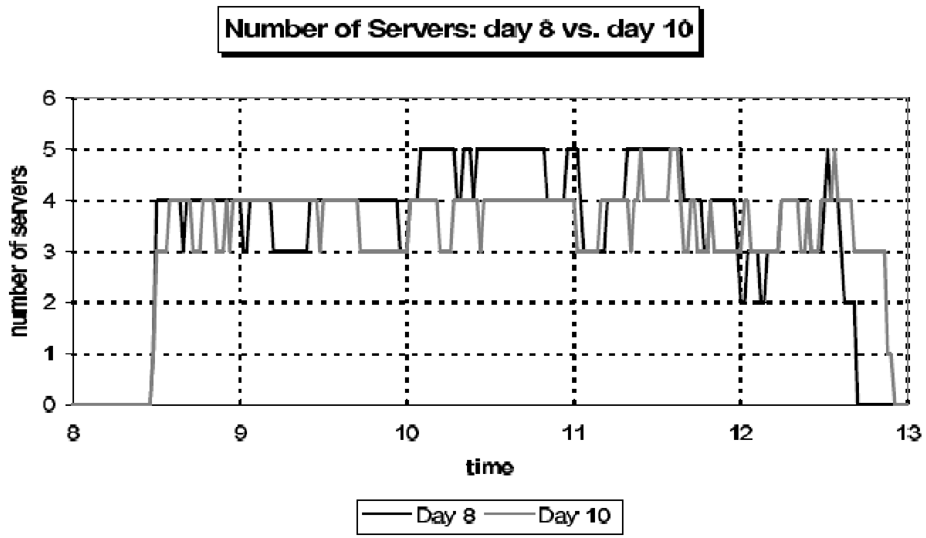


Case 2. Staffing (2)

Comparison between
two heavily loaded days

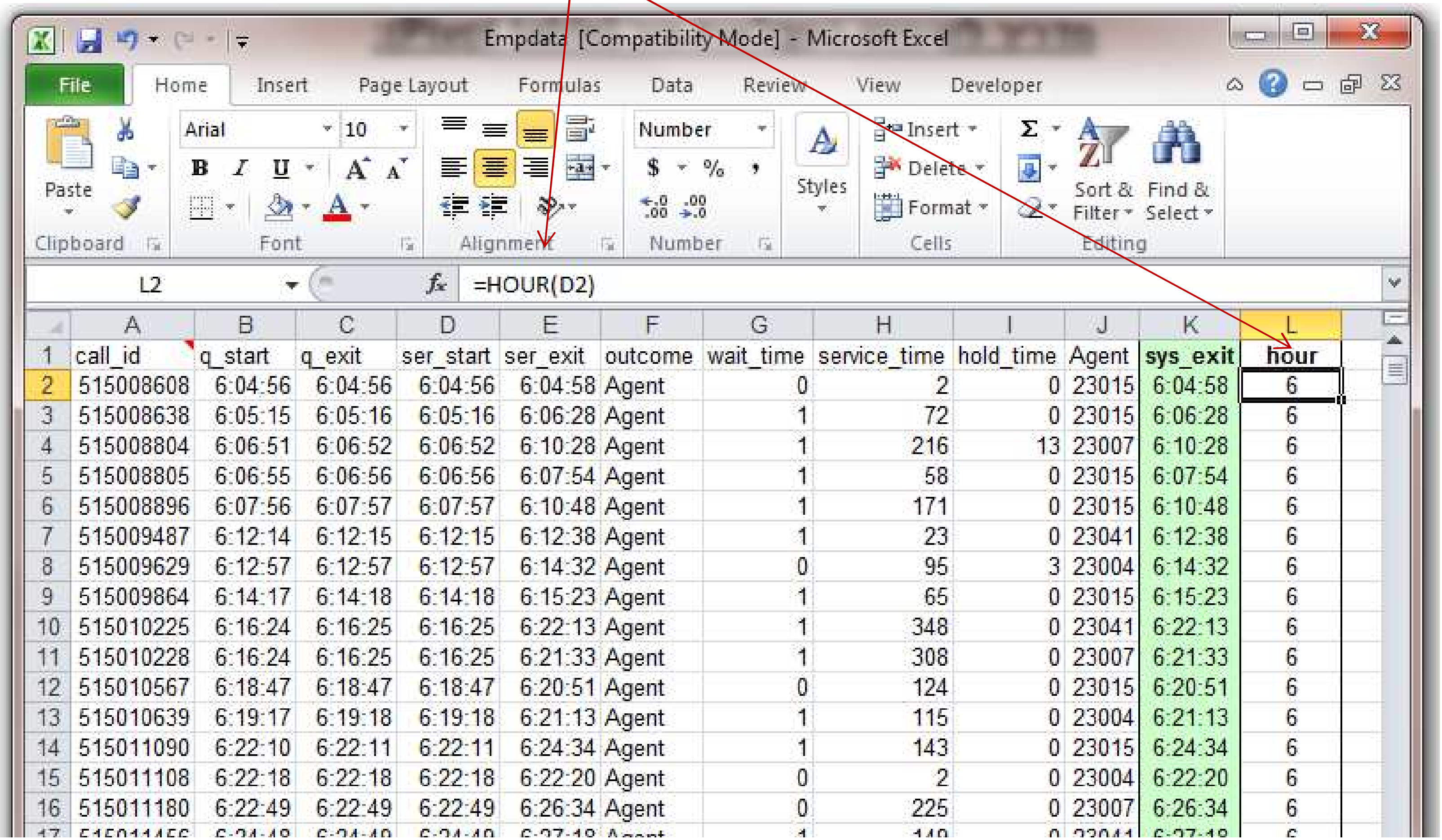






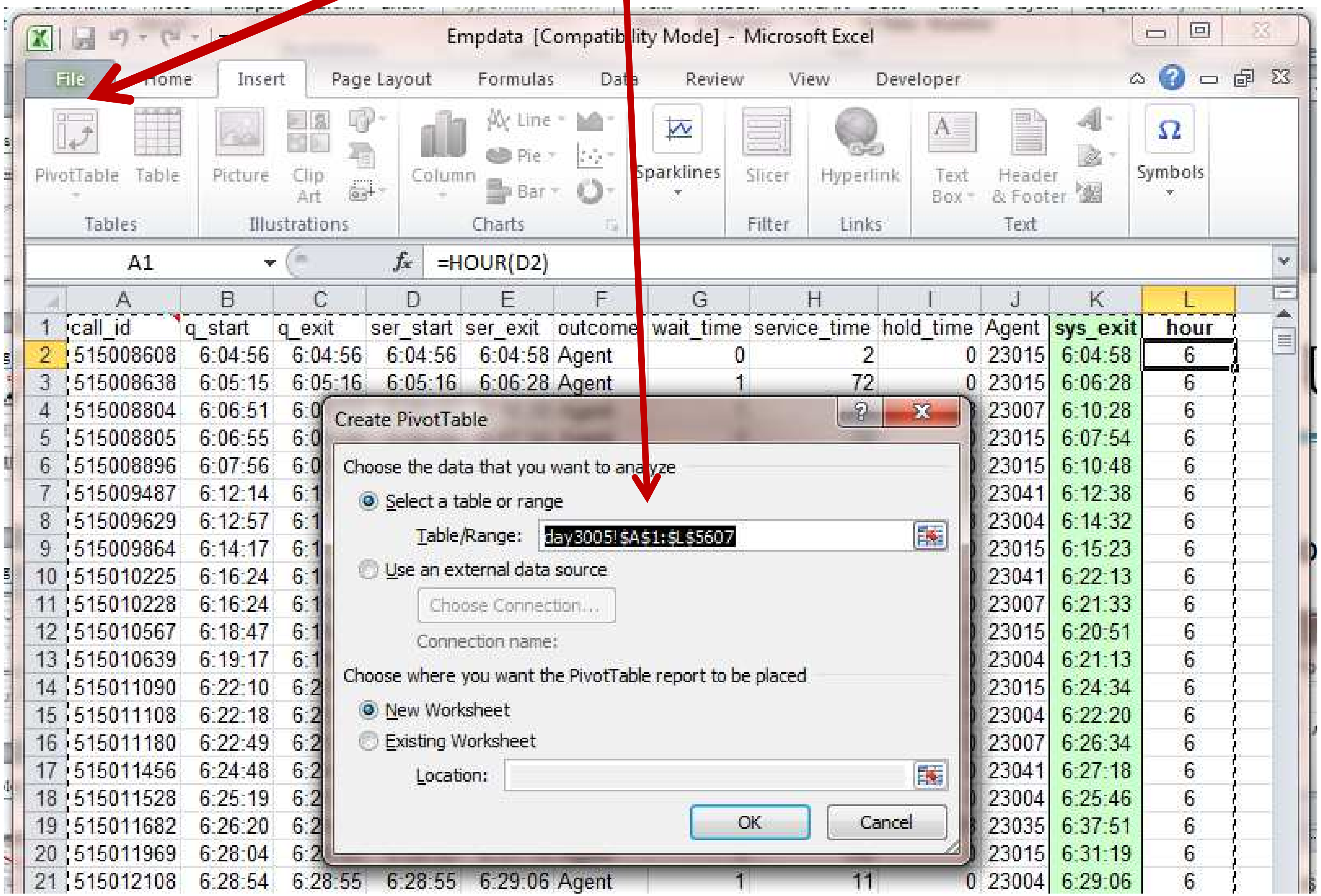
Using Pivot Table (1)

- Empdata.xls
  - Time functions: =hour(), =minute()



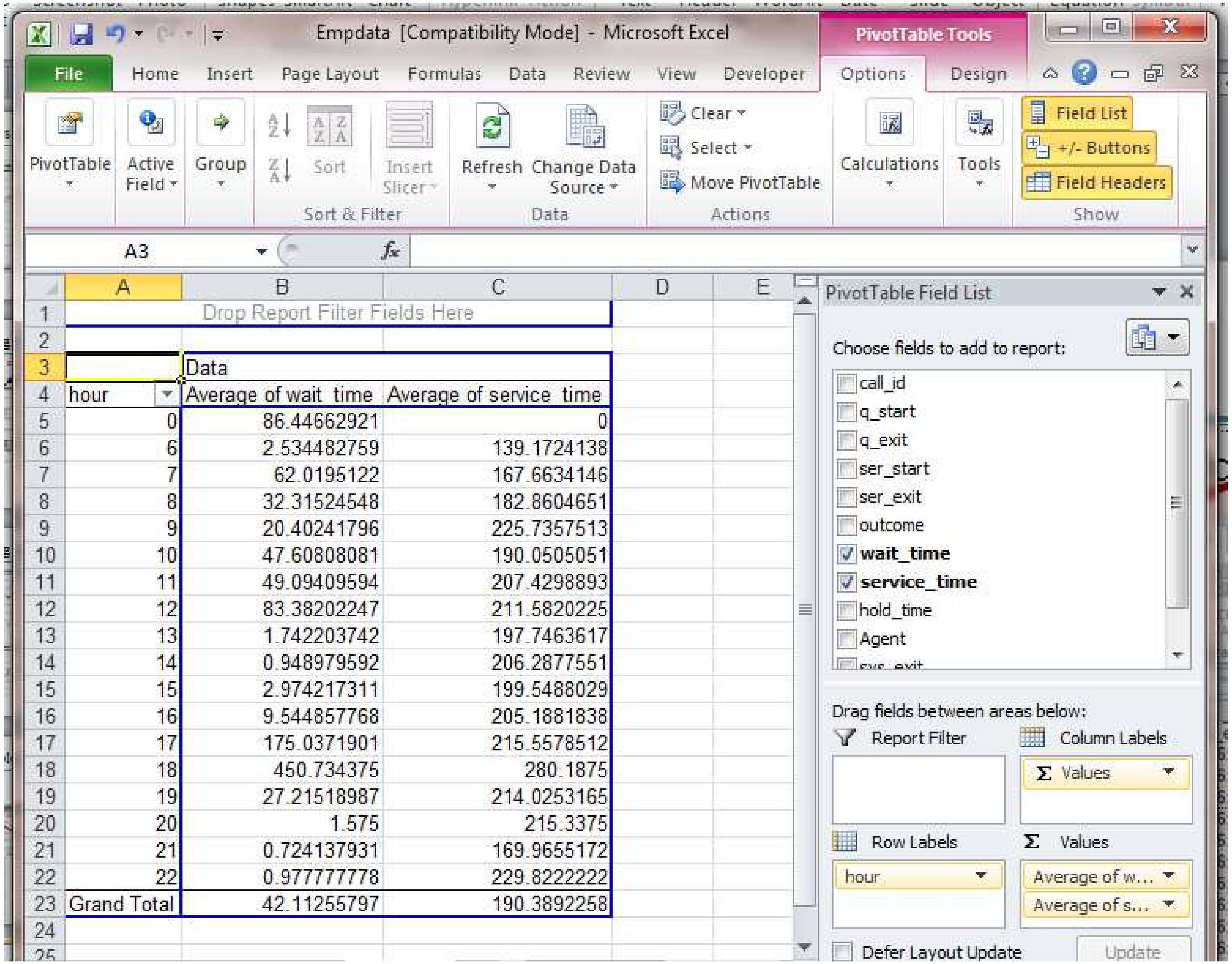
	A	B	C	D	E	F	G	H	I	J	K	L
1	call_id	q_start	q_exit	ser_start	ser_exit	outcome	wait_time	service_time	hold_time	Agent	sys_exit	hour
2	515008608	6:04:56	6:04:56	6:04:56	6:04:58	Agent	0	2	0	23015	6:04:58	6
3	515008638	6:05:15	6:05:16	6:05:16	6:06:28	Agent	1	72	0	23015	6:06:28	6
4	515008804	6:06:51	6:06:52	6:06:52	6:10:28	Agent	1	216	13	23007	6:10:28	6
5	515008805	6:06:55	6:06:56	6:06:56	6:07:54	Agent	1	58	0	23015	6:07:54	6
6	515008896	6:07:56	6:07:57	6:07:57	6:10:48	Agent	1	171	0	23015	6:10:48	6
7	515009487	6:12:14	6:12:15	6:12:15	6:12:38	Agent	1	23	0	23041	6:12:38	6
8	515009629	6:12:57	6:12:57	6:12:57	6:14:32	Agent	0	95	3	23004	6:14:32	6
9	515009864	6:14:17	6:14:18	6:14:18	6:15:23	Agent	1	65	0	23015	6:15:23	6
10	515010225	6:16:24	6:16:25	6:16:25	6:22:13	Agent	1	348	0	23041	6:22:13	6
11	515010228	6:16:24	6:16:25	6:16:25	6:21:33	Agent	1	308	0	23007	6:21:33	6
12	515010567	6:18:47	6:18:47	6:18:47	6:20:51	Agent	0	124	0	23015	6:20:51	6
13	515010639	6:19:17	6:19:18	6:19:18	6:21:13	Agent	1	115	0	23004	6:21:13	6
14	515011090	6:22:10	6:22:11	6:22:11	6:24:34	Agent	1	143	0	23015	6:24:34	6
15	515011108	6:22:18	6:22:18	6:22:18	6:22:20	Agent	0	2	0	23004	6:22:20	6
16	515011180	6:22:49	6:22:49	6:22:49	6:26:34	Agent	0	225	0	23007	6:26:34	6

Using Pivot Table (2)



Using Pivot Table (3)

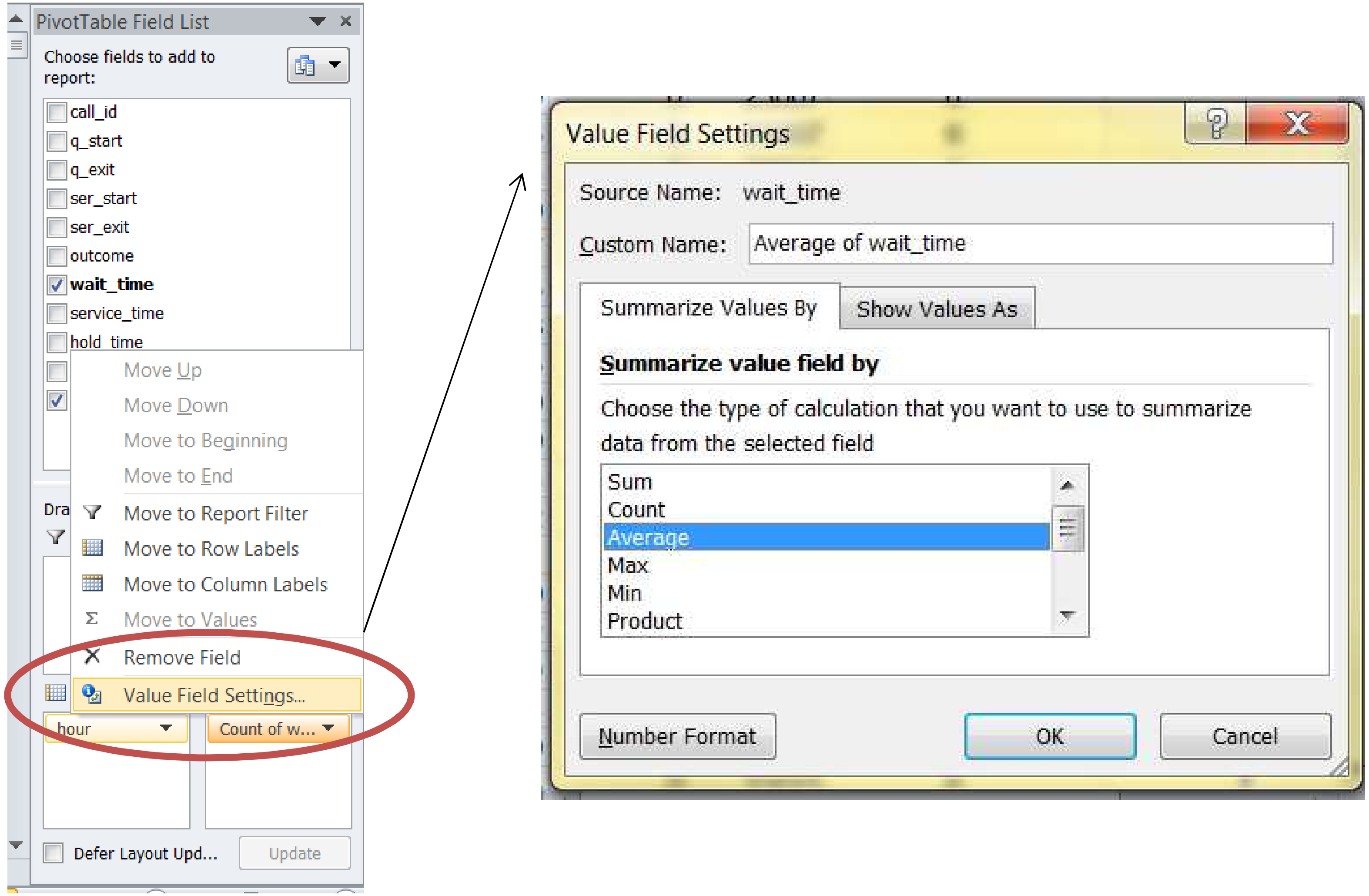
- Drag fields to Column Labels, Row Labels and Values



	Data	
hour	Average of wait_time	Average of service_time
0	86.44662921	0
6	2.534482759	139.1724138
7	62.0195122	167.6634146
8	32.31524548	182.8604651
9	20.40241796	225.7357513
10	47.60808081	190.0505051
11	49.09409594	207.4298893
12	83.38202247	211.5820225
13	1.742203742	197.7463617
14	0.948979592	206.2877551
15	2.974217311	199.5488029
16	9.544857768	205.1881838
17	175.0371901	215.5578512
18	450.734375	280.1875
19	27.21518987	214.0253165
20	1.575	215.3375
21	0.724137931	169.9655172
22	0.977777778	229.8222222
Grand Total	42.11255797	190.3892258

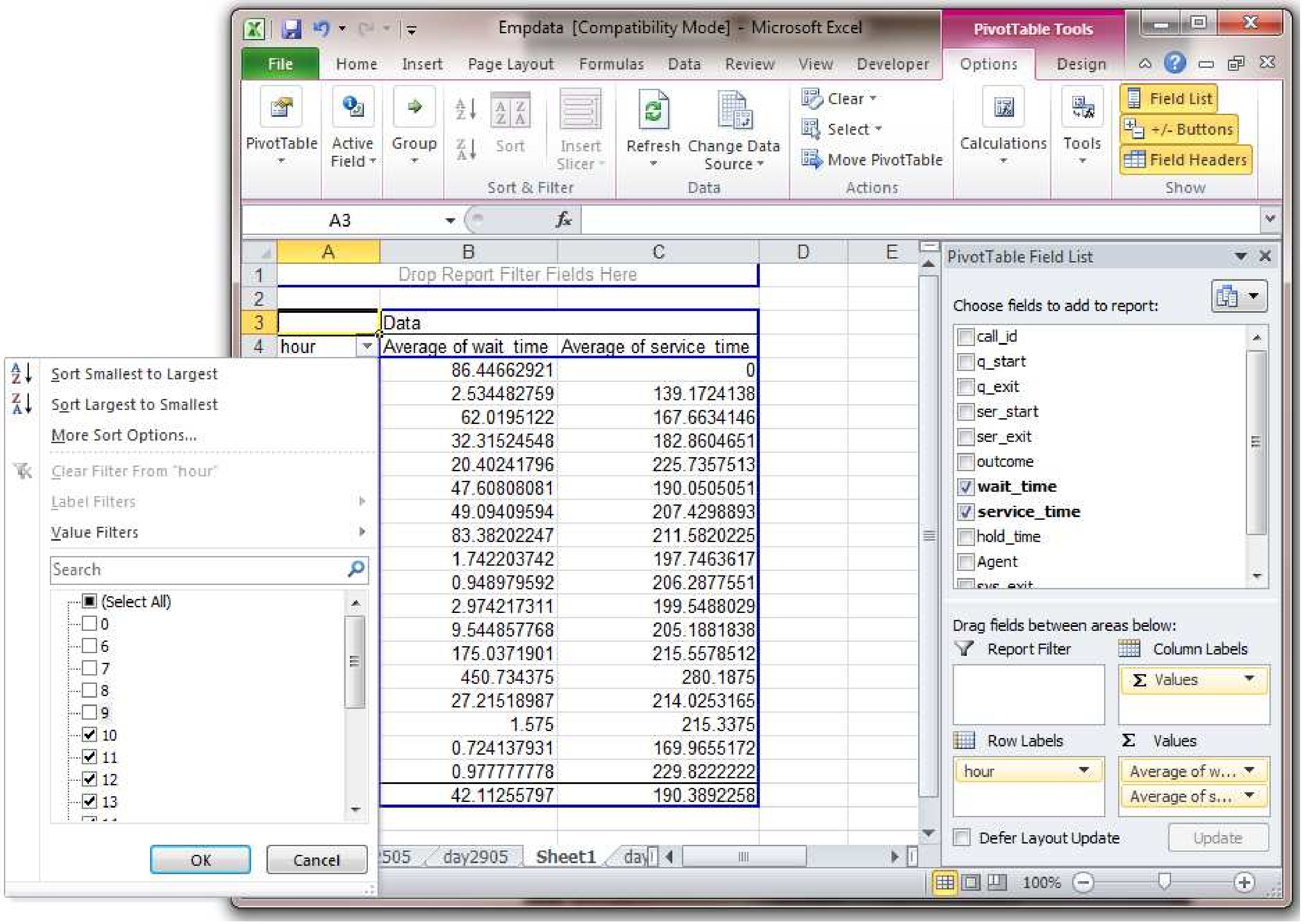
Using Pivot Table (4)

- Changing **Value Field Settings**: sum, count (default), average, max, min…



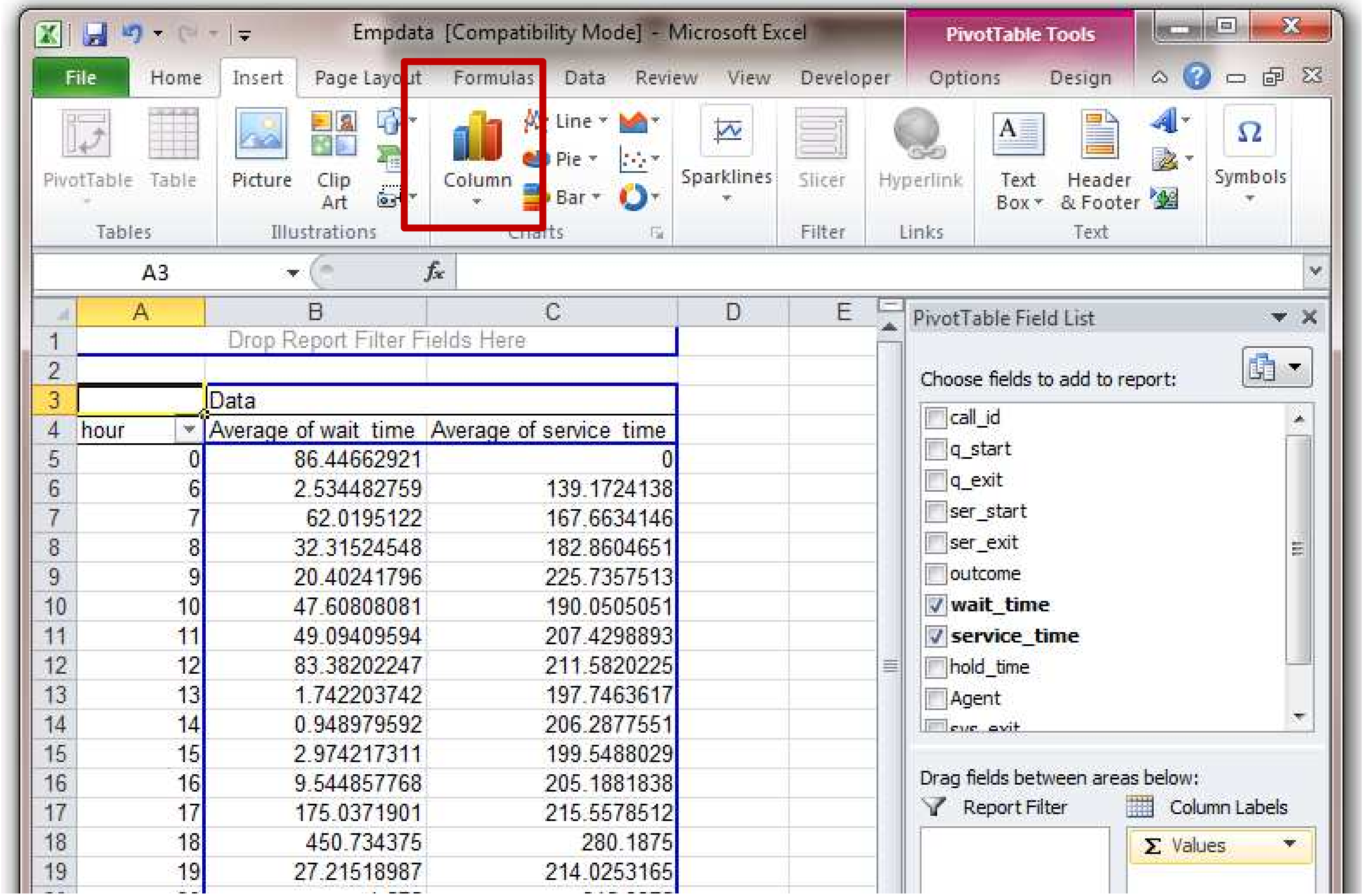
Using Pivot Table (5)

- Can use filters



Using Pivot Table (6)

- Drawing graphs using Pivot Table



Using Pivot Table (7)

- Drawing graphs using Pivot Table