

# Empirical Models

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- Case 1. Printing Shop Example (p.2-10)
- Case 2. Face-to-Face Service Operation Example (p. 11-20)
- Using Pivot Table Tool in EXCEL (p. 21-27)

# Case 1. Printing Shop Example

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- Jobs at a printing shop from 9AM to 12:20PM

| Customer | Arrival time | Depart queue | Depart system |
|----------|--------------|--------------|---------------|
| 1        | 9:03         | 9:03         | 9:23          |
| 2        | 9:10         | 9:23         | 9:55          |
| 3        | 9:40         | 9:55         | 10:35         |
| 4        | 10:05        | 10:35        | 10:50         |
| 5        | 10:15        | 10:50        | 11:20         |
| 6        | 10:40        | 11:20        | 11:35         |
| 7        | 11:50        | 11:50        | 12:20         |

- Q1** Calculate the average time spent in queue and in the system
- Q2** Plot the cumulative arrivals, cumulative departures from queue, and cumulative departures from the system
- Q3** For 9:00 - 12:20, determine the average number of customers in queue and in the system.
- Q4** From your answers to Q1 and Q3, verify Little's formula for the customers in queue and in the system in the time interval 9:00-12:20
- Q5** What would happen to the law, if you consider only the time interval 9:00-12:00?
- Q6** Determine the throughput and utilization over 9:00 - 12:20

# Case 1 - Q1

- **Q1:** Average time spent in queue and in the system

- Define

- $A^{-1}(n)$ : time of the  $n^{\text{th}}$  arrival
- $D_q^{-1}(n)$ : time of the  $n^{\text{th}}$  departure from queue
- $D_{\text{sys}}^{-1}(n)$ : time of the  $n^{\text{th}}$  departure from system
- $W_q(n)$ : time in queue, for the  $n^{\text{th}}$  customer to arrive.  $W_q(n) = D_q^{-1}(n) - A^{-1}(n)$
- $W_{\text{sys}}(n)$ : time in system, for the  $n^{\text{th}}$  customer to arrive.  $W_{\text{sys}}(n) = D_{\text{sys}}^{-1}(n) - A^{-1}(n)$

| Customer | Arrival time | Depart queue | Depart system |
|----------|--------------|--------------|---------------|
| 1        | 9:03         | 9:03         | 9:23          |
| 2        | 9:10         | 9:23         | 9:55          |
| 3        | 9:40         | 9:55         | 10:35         |
| 4        | 10:05        | 10:35        | 10:50         |
| 5        | 10:15        | 10:50        | 11:20         |
| 6        | 10:40        | 11:20        | 11:35         |
| 7        | 11:50        | 11:50        | 12:20         |

| n   | $W_q(n)$ | $W_{\text{sys}}(n)$ |
|-----|----------|---------------------|
| 1   | 0        | 20                  |
| 2   | 13       | 45                  |
| 3   | 15       | 55                  |
| 4   | 30       | 45                  |
| 5   | 35       | 64                  |
| 6   | 40       | 55                  |
| 7   | 0        | 30                  |
| Avg | 19 min   | 45 min              |

- $\overline{W}_q$ : average time spent in queue
- $\overline{W}_{\text{sys}}$ : average time spent in system (queue + service)
- $\overline{W}_{\text{service}}$ : average time spent in service ( $\overline{W}_{\text{sys}} - \overline{W}_q = 26 \text{ min}$ )

$\overline{W}_q$        $\overline{W}_{\text{sys}}$

# Case 1 – Q2

- **Q2**: Plot the cumulative arrivals, cumulative departures from queue, and cumulative departures from the system.

Define

- $A(t)$ : cumulative arrivals from time 0 to time  $t$
- $D_q(t)$ : cumulative departures from the queue from time 0 to time  $t$
- $D_{sys}(t)$ : cumulative departures from the system from time 0 to time  $t$

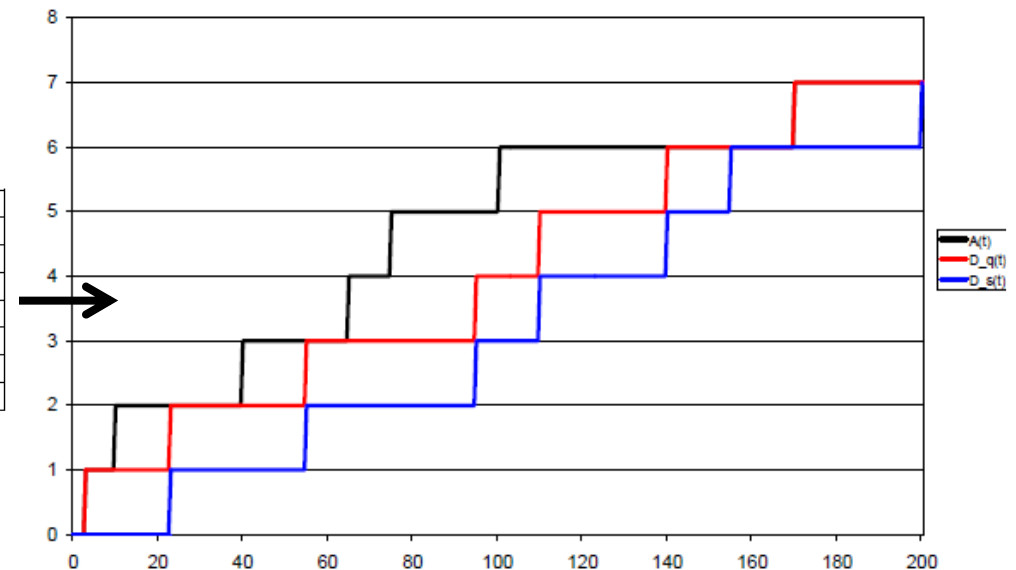
| min  | $A(t)$ | $D_q(t)$ | $D_{sys}(t)$ |
|------|--------|----------|--------------|
| 0    | 0      | 0        | 0            |
| 0.5  | 0      | 0        | 0            |
| 1    | 0      | 0        | 0            |
| 1.5  | 0      | 0        | 0            |
| 2    | 0      | 0        | 0            |
| 2.5  | 0      | 0        | 0            |
| 3    | 1      | 1        | 0            |
| 3.5  | 1      | 1        | 0            |
| :    |        |          |              |
| 9    | 1      | 1        | 0            |
| 9.5  | 1      | 1        | 0            |
| 10   | 2      | 1        | 0            |
| 10.5 | 2      | 1        | 0            |
| :    |        |          |              |
| 21   | 2      | 1        | 0            |
| 21.5 | 2      | 1        | 0            |
| 22   | 2      | 2        | 1            |
| 22.5 | 2      | 2        | 1            |

# Case 1 – Q2 continued

- **Q2:** Plot the cumulative arrivals, cumulative departures from queue, and cumulative departures from the system.

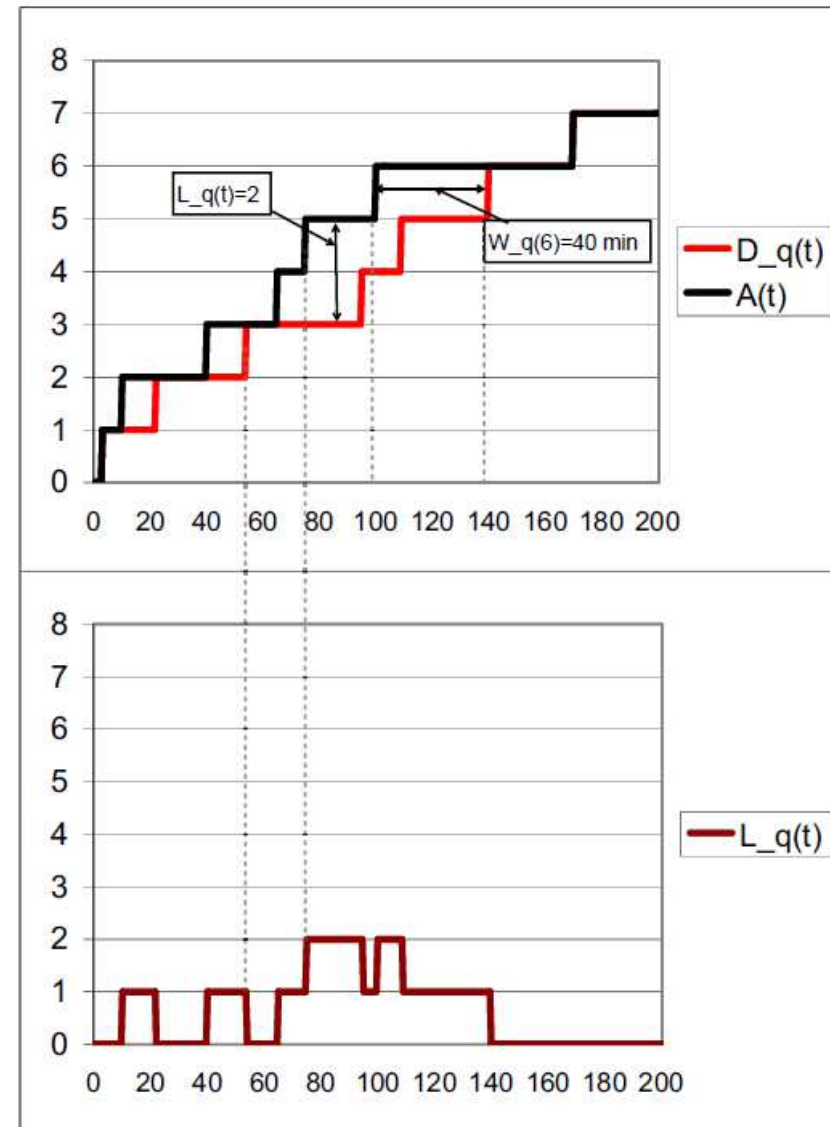
$$A(t) \geq D_q(t) \geq D_{sys}(t).$$

| Customer | Arrival time | Depart queue | Depart system |
|----------|--------------|--------------|---------------|
| 1        | 9:03         | 9:03         | 9:23          |
| 2        | 9:10         | 9:23         | 9:55          |
| 3        | 9:40         | 9:55         | 10:35         |
| 4        | 10:05        | 10:35        | 10:50         |
| 5        | 10:15        | 10:50        | 11:20         |
| 6        | 10:40        | 11:20        | 11:35         |
| 7        | 11:50        | 11:50        | 12:20         |



# Case 1 – Q3

- **Q3** For 9:00 - 12:20, determine the average number of customers in queue and in the system.
- Define
  - $L_q(t)$ : number of customers in the queue at time  $t$ .  $L_q(t) = A(t) - D_q(t)$ .
  - $L_{sys}(t)$ : number of customers in the system at time  $t$ .  $L_{sys}(t) = A(t) - D_{sys}(t)$ .
- $\bar{L}_q = \frac{\int_a^b L_q(t) dt}{b-a}$ : average queue length
  - The numerator is the area below the graph of  $L_q(t)$
- $\bar{L}_{sys} = \frac{\int_a^b L_{sys}(t) dt}{b-a}$ : average number of customers in the system



# Case 1 – Q3 continued

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- Then,
- $\bar{L}_q = \frac{\int_0^{200} L_q(t) dt}{200} = \frac{(13+15+10+5+30) \times 1 + (20+10) \times 2}{200} = 0.6651 \text{ cust}$
- $\bar{L}_{sys} = \frac{\int_0^{200} L_{sys}(t) dt}{200}$   
 $= \frac{(7+17+10+15+30) \times 1 + (13+15+10+5+30) \times 2 + (20+10) \times 3}{200} = 1.575 \text{ cust}$

# Case 1 – Q4

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- **Q4:** Verify Little's formula for the customers in the time interval 9:00-12:20
  - We have
    - $\lambda = \frac{7}{300} = 0.035 \frac{\text{cust}}{\text{min}}$
    - $W_q = 19 \text{ min}$  (from Q1)
    - $W_{\text{sys}} = 45 \text{ min}$  (from Q1)
- $\hat{L}_q^{[9:00,12:20]} = 0.035 \frac{\text{cust}}{\text{min}} \times 19 \text{ mins} = \mathbf{0.665 \text{ cust}}$
- same as the answer to Q3
- $\hat{L}_{\text{sys}}^{[9:00,12:20]} = 0.035 \frac{\text{cust}}{\text{min}} \times 45 \text{ mins} = \mathbf{1.575 \text{ cust}}$
- same as the answer to Q3

# Case 1 – Q5

- **Q5:** Does Little's formula hold for time interval 9:00-12:00?

- We have

- $\lambda = \frac{7}{300} = 0.035 \frac{\text{cust}}{\text{min}}$

- $\bar{W}_q = \frac{0+13+15+30+35+40+0}{7} = 19 \text{ minutes}$

- $\bar{W}_{\text{sys}} = \frac{20+45+55+45+64+55+10}{7} = 42.14 \text{ minutes}$

- $\bar{L}_q = \frac{\int_0^{180} L_q(t) dt}{180} = \frac{(13+15+10+5+10) \times 1 + (20+10) \times 2}{180} = 0.6279 \text{ cust}$

- $\bar{L}_{\text{sys}} = \frac{\int_0^{180} L_{\text{sys}}(t) dt}{180}$   
 $= \frac{(7 + 17 + 10 + 15 + 10) \cdot 1 + (13 + 15 + 10 + 5 + 30) \cdot 2 + (20 + 10) \cdot 3}{180} = 1.6389 \text{ cust}$

→  $\hat{L}_q^{[9:00,12:00]} = 0.035 \frac{\text{cust}}{\text{min}} \times 19 \text{ mins} = 0.665 \text{ cust}$

→  $\hat{L}_{\text{sys}}^{[9:00,12:00]} = 0.035 \frac{\text{cust}}{\text{min}} \times 42.14 \text{ mins} = 1.475 \text{ cust}$

**Little's law doesn't hold**

# Case 1 – Q6

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- Q6: Throughput and utilization over 9:00 - 12:20

- Throughput rate

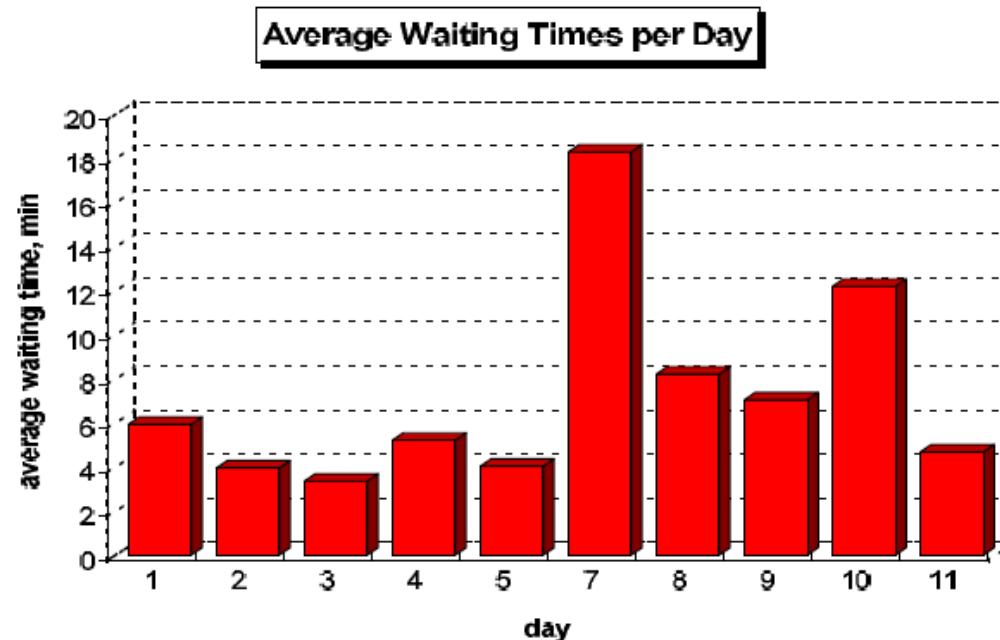
- $\frac{7 \text{ arrivals}}{200 \text{ minutes}} = 0.035 \text{ per minute}$

- Utilization

- $\rho = \frac{\text{busy time}}{200} = \frac{200-3-15}{200} = 0.91 \text{ (91\%)}$

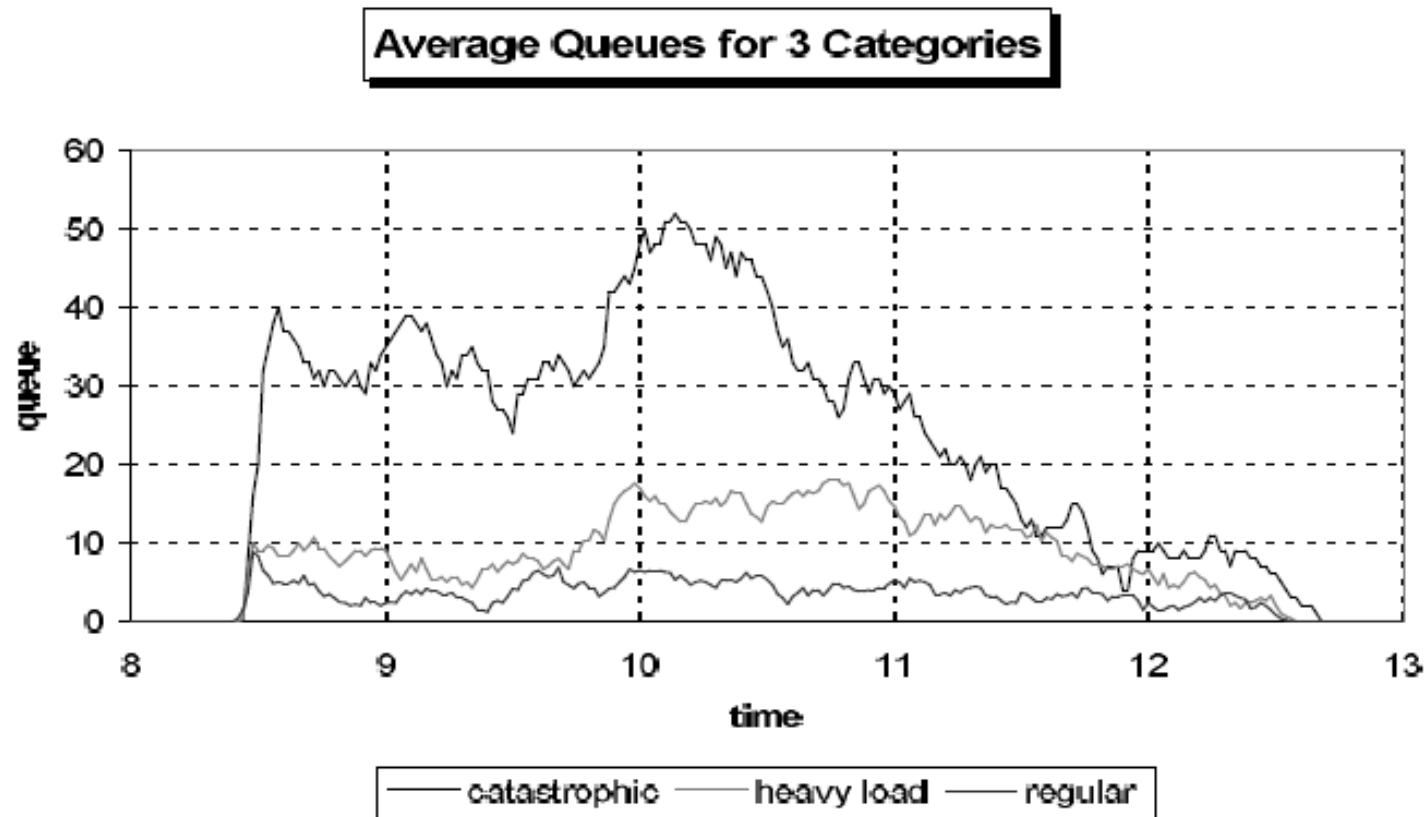
# Case 2. Face-to-Face Service of a Bank

- Data from **12 days** of work in a Face-to-Face service of a bank Several servers (maximum **5**) work simultaneously at a **single** station
- Working days were divided into three categories (Data from **Fridays (day 6, 12)** is not considered because they are different from the others)
  - Catastrophic day: day 7
  - Heavily loaded days: days 8, 9 and 10
  - Regular days: days 1, 2, 3, 4, 5 and 11



## Case 2. Queues (1)

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- Common features and queue pattern for each day?

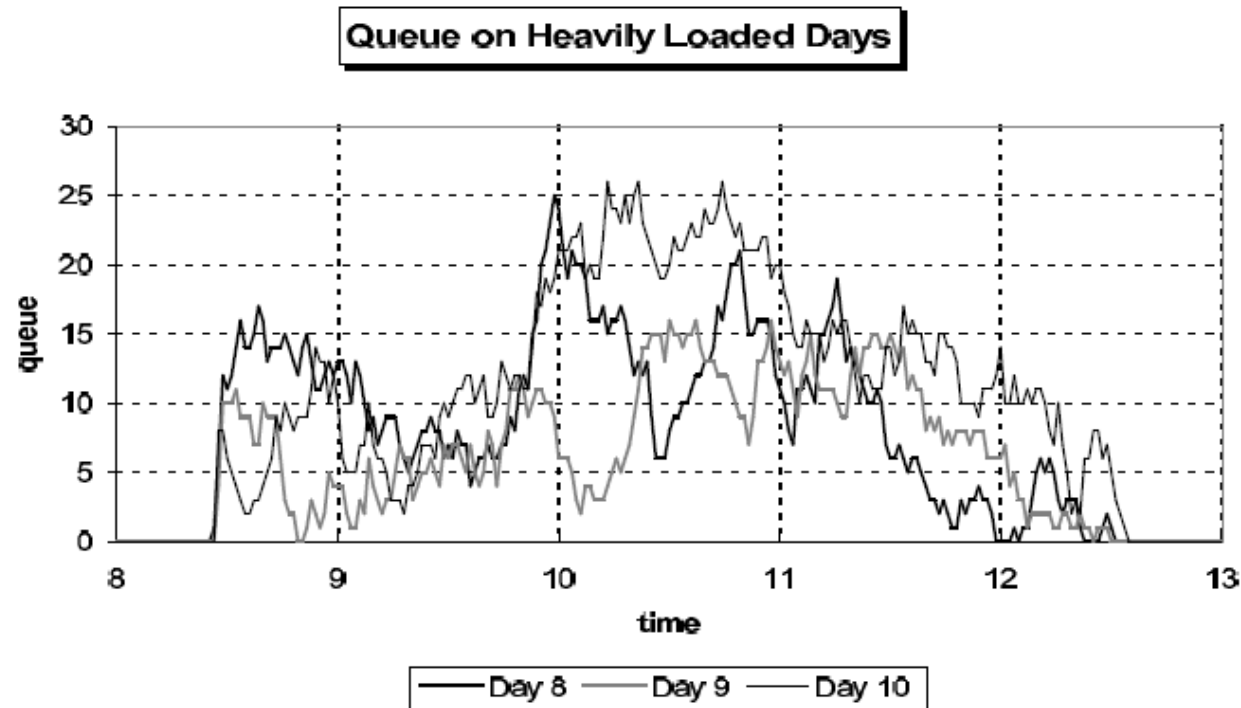
# Case 2. Queues (2)

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- Common features for different categories.
  - Sharp growth of the queues at the start of the day
    - At 8:30AM, the bank opens and there are people waiting at the door
  - Queue decrease before 9:30
  - Queue growth after 10:00
  - Gradual decrease to zero at the end of the day
- Average queue pattern for every category:
  - Catastrophic day (1 day)
    - Sharp increase when the day starts (40 customers shortly after 8:30)
    - Until 9:30, the queue goes down to 25 customers and then grows rapidly again
    - Approximately at 10:10, we get the record queue for all days of more than 50 customers
    - Then the queue gradually decreases to zero at the end of the day

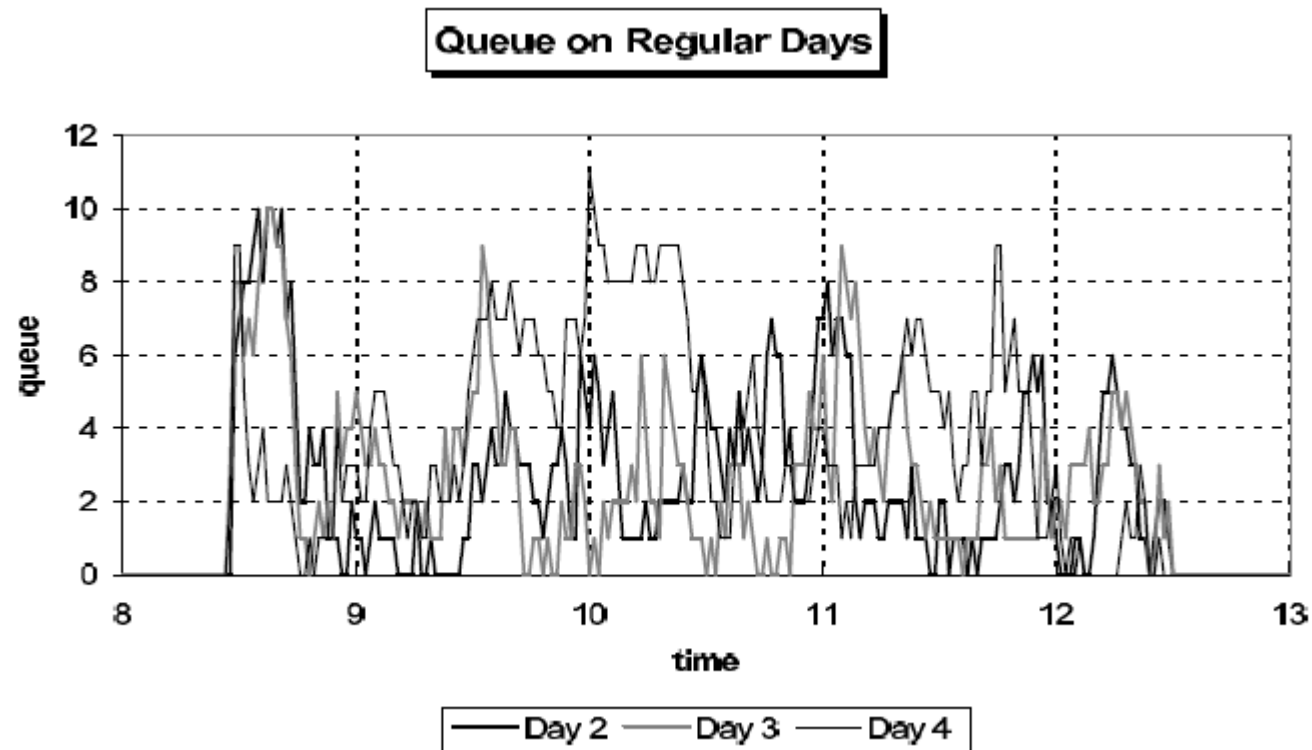
## Case 2. Queues (3)

- Heavily loaded days (Average of 3 days)
  - A sharp increase to 10 customers at the beginning of the day, and then oscillates between 5 and 10 customers until 9:45
  - After 11:00 the queue slowly decreases to zero
  - Sample path of each day:

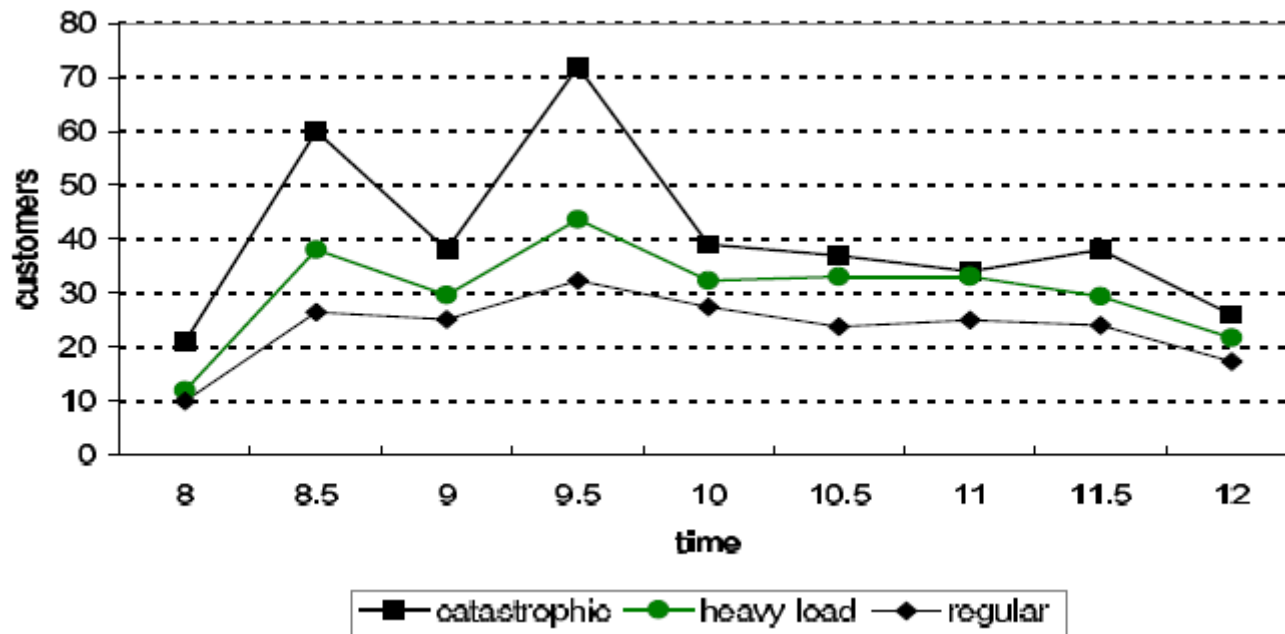


## Case 2. Queues (4)

- Regular days (Average of 6 days)
  - jumps to approximately 10 at the beginning
  - decreases close to zero before 9:30
  - over almost the whole day, oscillates in “steady state” near 5
  - Sample path of each day:

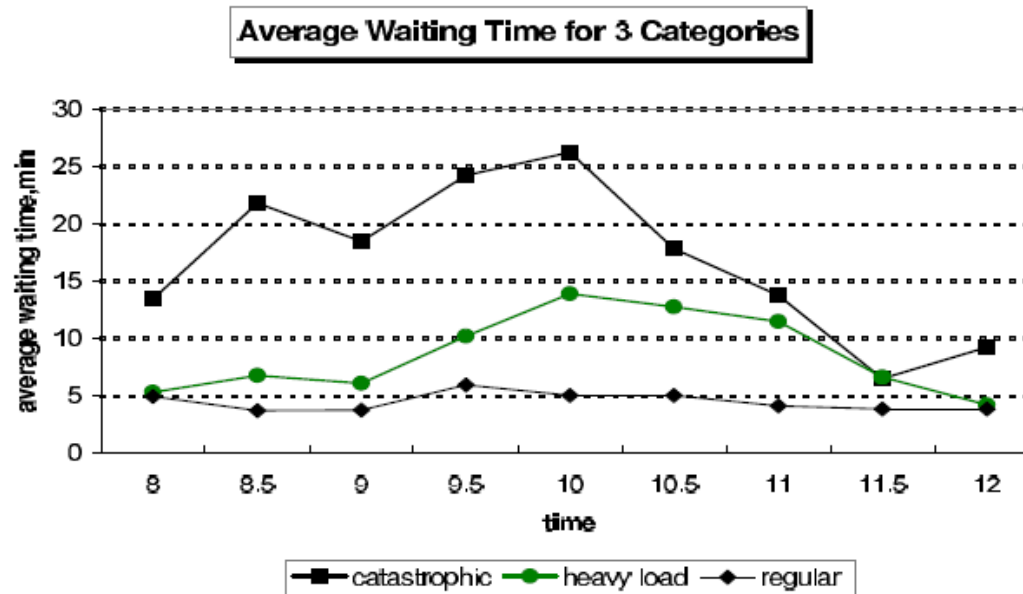


## Case 2. Arrival Rates



- We have maximum arrival rate at the same time interval (9:30–10:00) for all categories
- The maxima are “steeper” for the categories with a large arrival rate.
- **Catastrophic day** has two sharp peaks of arrivals: 60 customer between 8:30 and 9:00 and more than 70 customers between 9:30 and 10:00. Busy morning.
- **Heavily loaded days** has two maximums at 8:30–9:00 and 9:30–10:00 (approximately 40 customers). For other time intervals, except for the first and the last, the arrival rate is nearly constant (30 customers).
- For **regular days**, the variability of the average arrival rate is small.

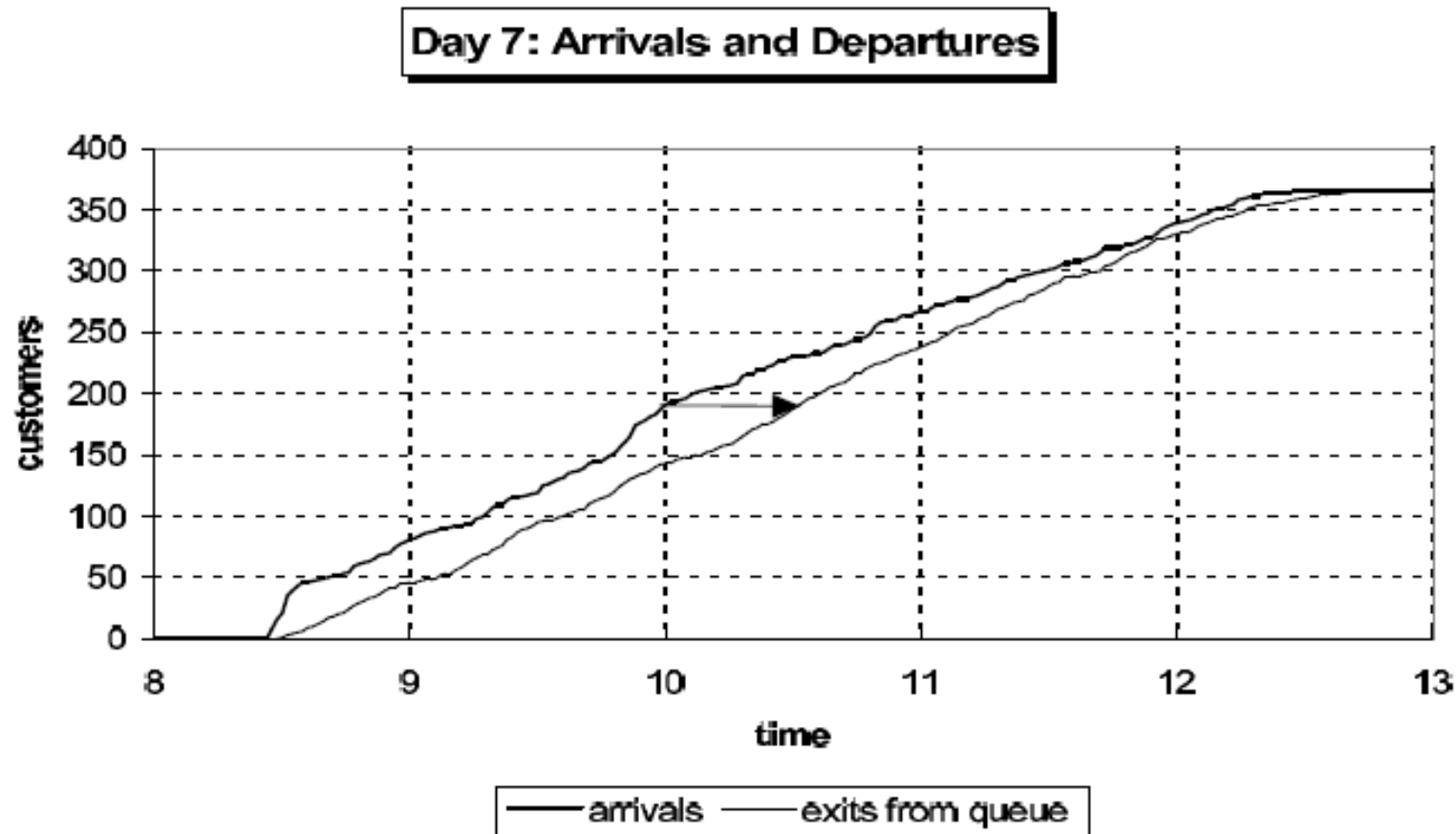
# Case 2. Waiting Times



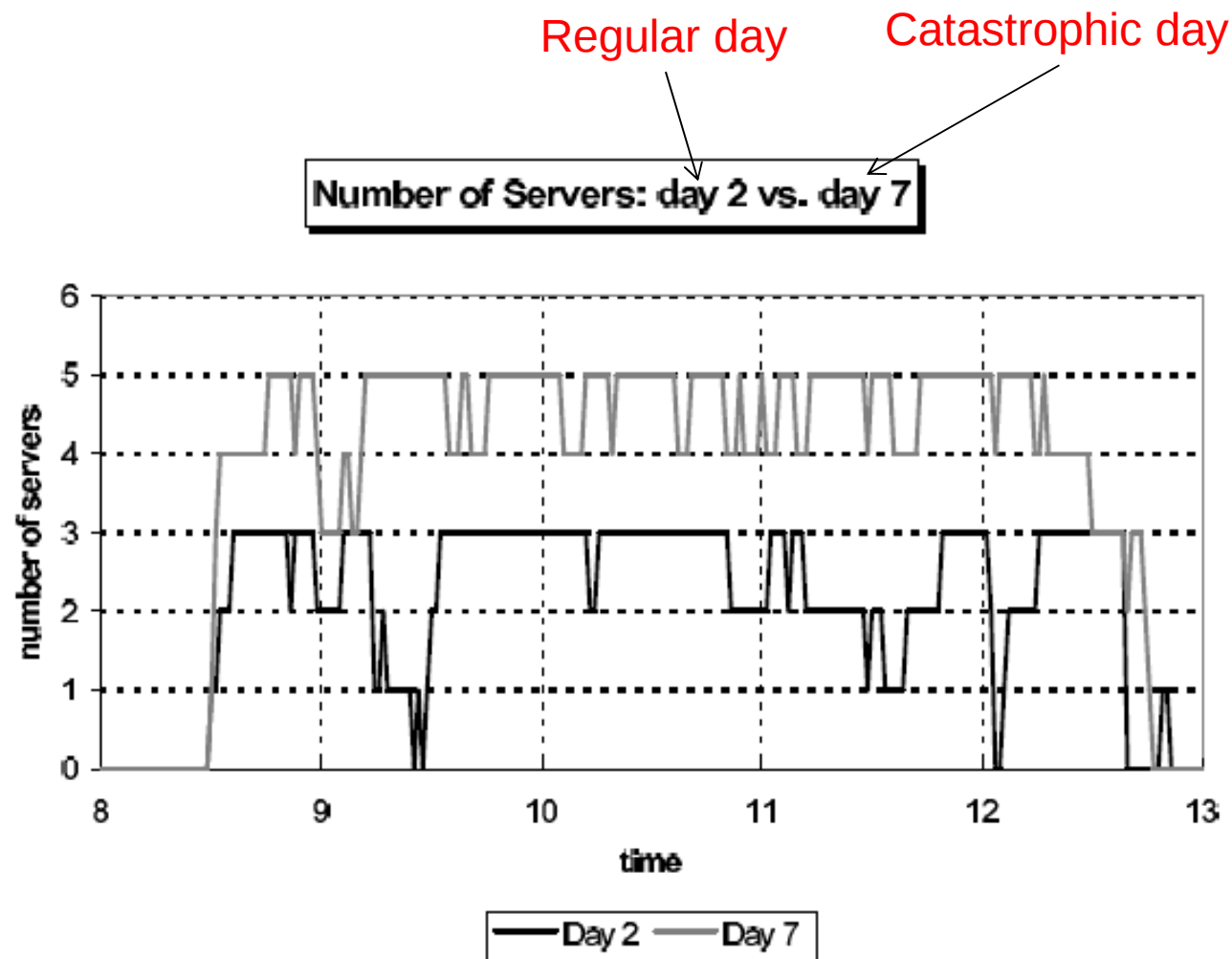
- For catastrophic and heavily loaded days, maximum value at [10:00, 10:30]
  - For a heavily loaded queueing system, the peak of the waiting time usually comes *after* the peak of the arrival rate (Slide 16)
- The patterns of the waiting times follow the **patterns of queues** (slide 12)
- For **Catastrophic day**, rapid decrease after 10:30
- For **heavily loaded days**, the average waiting time is relatively small until 9:30 (5-6 minutes). In the time interval 9:30-11:30 it is over 10 minutes with maximum between 10 and 10:30 (approximately 14 minutes). After 11:30 the waiting times are small.
- For **regular days**, the average waiting time does not change considerably (oscillates around 5 minutes) with time. The maximum takes place in the interval 9:30-10:00 .

## Case 2. Waiting Times - Estimate

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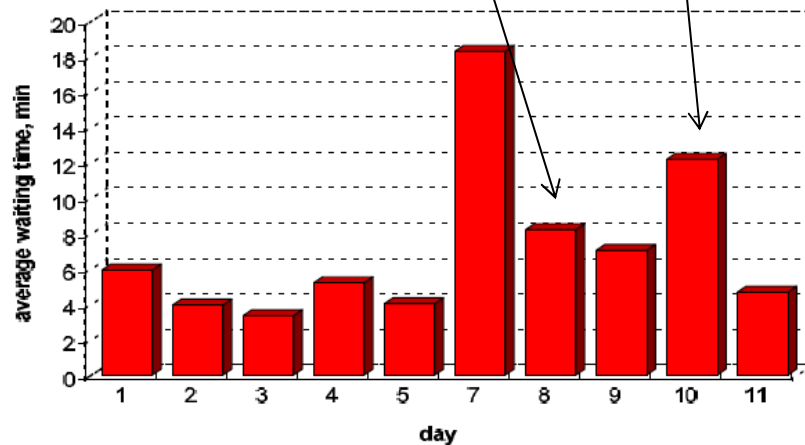
# Case 2. Staffing (1)



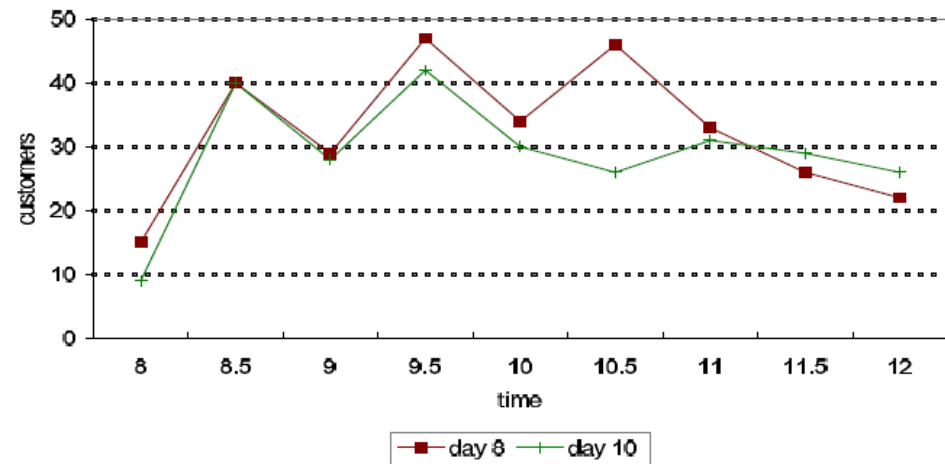
# Case 2. Staffing (2)

Comparison between two heavily loaded days

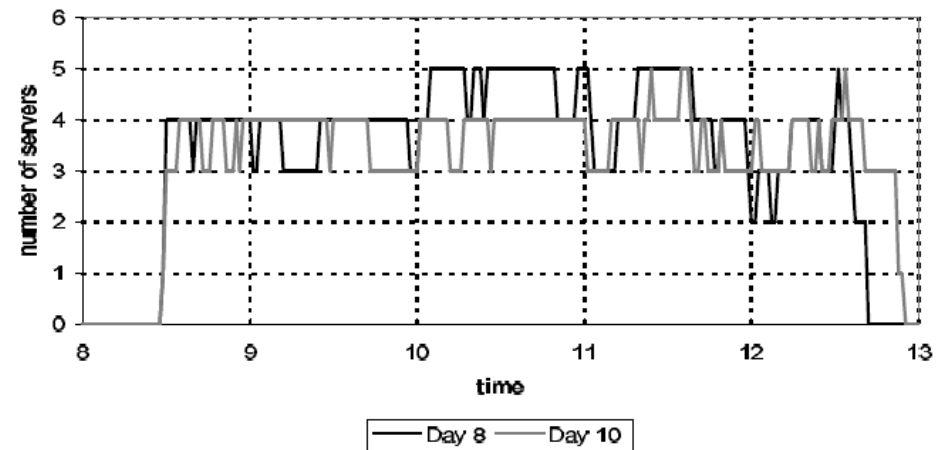
Average Waiting Times per Day



Arrival Rate: Day 8 vs. Day 10

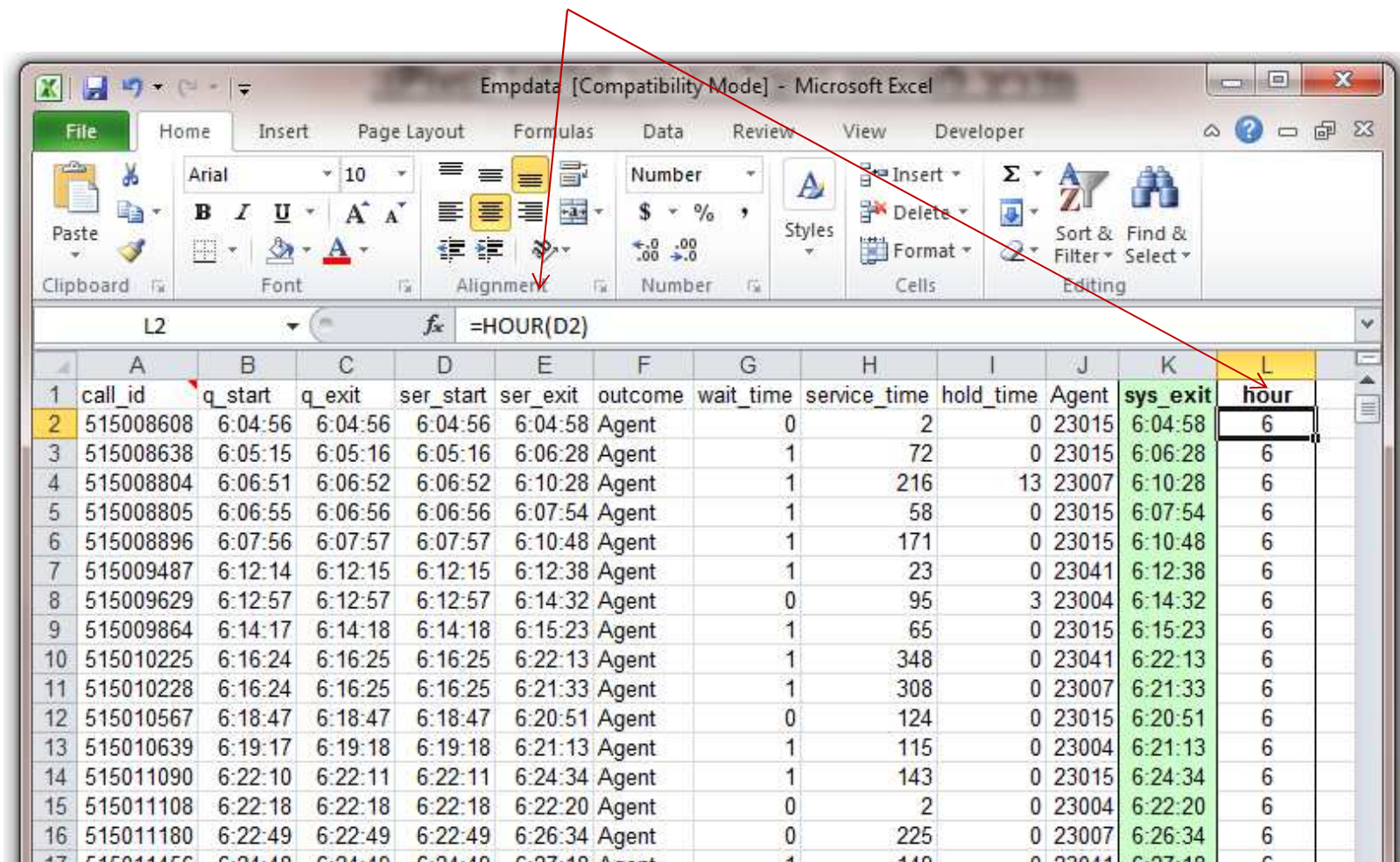


Number of Servers: day 8 vs. day 10



# Using Pivot Table (1)

- Empdata.xls
  - Time functions: =hour(), =minute()



Empdata [Compatibility Mode] - Microsoft Excel

File Home Insert Page Layout Formulas Data Review View Developer

Clipboard Font Alignment Number Styles Cells Editing

L2 =HOUR(D2)

|    | A         | B       | C       | D         | E        | F       | G         | H            | I         | J     | K        | L    |
|----|-----------|---------|---------|-----------|----------|---------|-----------|--------------|-----------|-------|----------|------|
| 1  | call_id   | q_start | q_exit  | ser_start | ser_exit | outcome | wait_time | service_time | hold_time | Agent | sys_exit | hour |
| 2  | 515008608 | 6:04:56 | 6:04:56 | 6:04:56   | 6:04:58  | Agent   | 0         | 2            | 0         | 23015 | 6:04:58  | 6    |
| 3  | 515008638 | 6:05:15 | 6:05:16 | 6:05:16   | 6:06:28  | Agent   | 1         | 72           | 0         | 23015 | 6:06:28  | 6    |
| 4  | 515008804 | 6:06:51 | 6:06:52 | 6:06:52   | 6:10:28  | Agent   | 1         | 216          | 13        | 23007 | 6:10:28  | 6    |
| 5  | 515008805 | 6:06:55 | 6:06:56 | 6:06:56   | 6:07:54  | Agent   | 1         | 58           | 0         | 23015 | 6:07:54  | 6    |
| 6  | 515008896 | 6:07:56 | 6:07:57 | 6:07:57   | 6:10:48  | Agent   | 1         | 171          | 0         | 23015 | 6:10:48  | 6    |
| 7  | 515009487 | 6:12:14 | 6:12:15 | 6:12:15   | 6:12:38  | Agent   | 1         | 23           | 0         | 23041 | 6:12:38  | 6    |
| 8  | 515009629 | 6:12:57 | 6:12:57 | 6:12:57   | 6:14:32  | Agent   | 0         | 95           | 3         | 23004 | 6:14:32  | 6    |
| 9  | 515009864 | 6:14:17 | 6:14:18 | 6:14:18   | 6:15:23  | Agent   | 1         | 65           | 0         | 23015 | 6:15:23  | 6    |
| 10 | 515010225 | 6:16:24 | 6:16:25 | 6:16:25   | 6:22:13  | Agent   | 1         | 348          | 0         | 23041 | 6:22:13  | 6    |
| 11 | 515010228 | 6:16:24 | 6:16:25 | 6:16:25   | 6:21:33  | Agent   | 1         | 308          | 0         | 23007 | 6:21:33  | 6    |
| 12 | 515010567 | 6:18:47 | 6:18:47 | 6:18:47   | 6:20:51  | Agent   | 0         | 124          | 0         | 23015 | 6:20:51  | 6    |
| 13 | 515010639 | 6:19:17 | 6:19:18 | 6:19:18   | 6:21:13  | Agent   | 1         | 115          | 0         | 23004 | 6:21:13  | 6    |
| 14 | 515011090 | 6:22:10 | 6:22:11 | 6:22:11   | 6:24:34  | Agent   | 1         | 143          | 0         | 23015 | 6:24:34  | 6    |
| 15 | 515011108 | 6:22:18 | 6:22:18 | 6:22:18   | 6:22:20  | Agent   | 0         | 2            | 0         | 23004 | 6:22:20  | 6    |
| 16 | 515011180 | 6:22:49 | 6:22:49 | 6:22:49   | 6:26:34  | Agent   | 0         | 225          | 0         | 23007 | 6:26:34  | 6    |
| 17 | 515011156 | 6:24:49 | 6:24:49 | 6:24:49   | 6:27:48  | Agent   | 1         | 110          | 0         | 23041 | 6:27:48  | 6    |

# Using Pivot Table (2)

The screenshot shows the Microsoft Excel interface with the 'Insert' tab selected. The 'PivotTable' button is highlighted in the 'Tables' group. A red arrow points from the title 'Using Pivot Table (2)' to this button. Another red arrow points from the 'PivotTable' button to the 'Table/Range' field in the 'Create PivotTable' dialog box, which contains the text 'day3005!\$A\$1:\$L\$5607'.

The background spreadsheet shows a table with columns: call\_id, q\_start, q\_exit, ser\_start, ser\_exit, outcome, wait\_time, service\_time, hold\_time, Agent, sys\_exit, and hour. The data is organized by Agent and time.

**Create PivotTable**

Choose the data that you want to analyze

- ☒ Select a table or range
- Table/Range: day3005!\$A\$1:\$L\$5607
- ☐ Use an external data source
- Choose Connection...
- Connection name:

Choose where you want the PivotTable report to be placed

- ☒ New Worksheet
- ☐ Existing Worksheet
- Location:

OK Cancel

# Using Pivot Table (3)

- Drag fields to Column Labels, Row Labels and Values

The screenshot displays the Microsoft Excel interface with a PivotTable and the PivotTable Field List task pane. The PivotTable is located in the range A3:C23, with 'hour' as the Row Label, and 'Average of wait time' and 'Average of service time' as Column Labels. The PivotTable Field List task pane is open on the right, showing the 'wait\_time' and 'service\_time' fields selected for the Values area.

**PivotTable Data:**

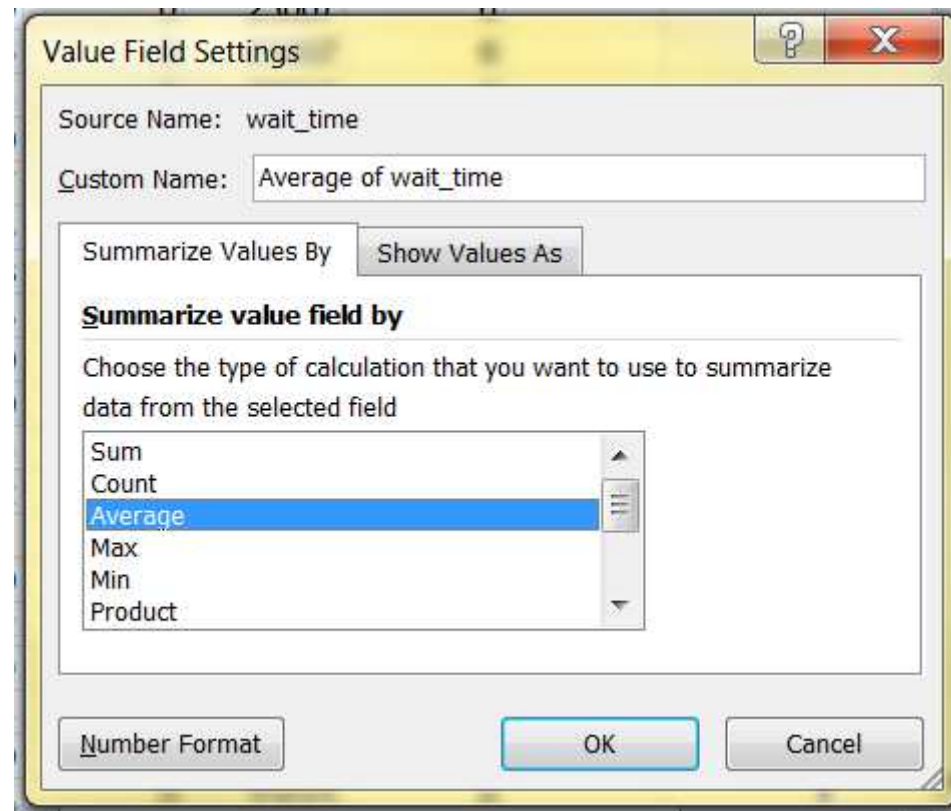
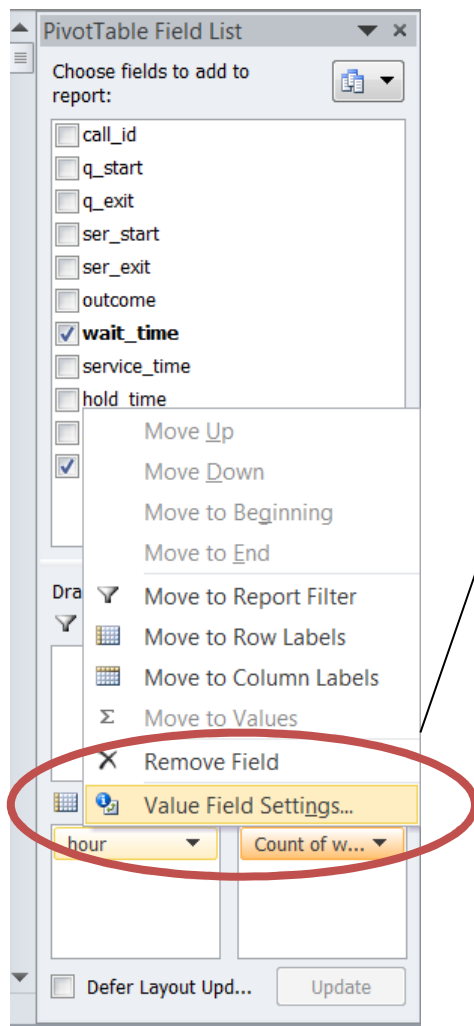
| hour        | Average of wait time | Average of service time |
|-------------|----------------------|-------------------------|
| 0           | 86.44662921          | 0                       |
| 6           | 2.534482759          | 139.1724138             |
| 7           | 62.0195122           | 167.6634146             |
| 8           | 32.31524548          | 182.8604651             |
| 9           | 20.40241796          | 225.7357513             |
| 10          | 47.60808081          | 190.0505051             |
| 11          | 49.09409594          | 207.4298893             |
| 12          | 83.38202247          | 211.5820225             |
| 13          | 1.742203742          | 197.7463617             |
| 14          | 0.948979592          | 206.2877551             |
| 15          | 2.974217311          | 199.5488029             |
| 16          | 9.544857768          | 205.1881838             |
| 17          | 175.0371901          | 215.5578512             |
| 18          | 450.734375           | 280.1875                |
| 19          | 27.21518987          | 214.0253165             |
| 20          | 1.575                | 215.3375                |
| 21          | 0.724137931          | 169.9655172             |
| 22          | 0.977777778          | 229.8222222             |
| Grand Total | 42.11255797          | 190.3892258             |

**PivotTable Field List:**

- Choose fields to add to report:
  - ☐ call\_id
  - ☐ q\_start
  - ☐ q\_exit
  - ☐ ser\_start
  - ☐ ser\_exit
  - ☐ outcome
  - ☒ wait\_time
  - ☒ service\_time
  - ☐ hold\_time
  - ☐ Agent
  - ☐ eve\_exit
- Drag fields between areas below:
  - Report Filter: (empty)
  - Column Labels: (empty)
  - Row Labels: hour
  - Values: Average of wait time, Average of service time
- ☐ Defer Layout Update
- Update

# Using Pivot Table (4)

- Changing **Value Field Settings**: sum, count (default), average, max, min...



# Using Pivot Table (5)

- Can use filters

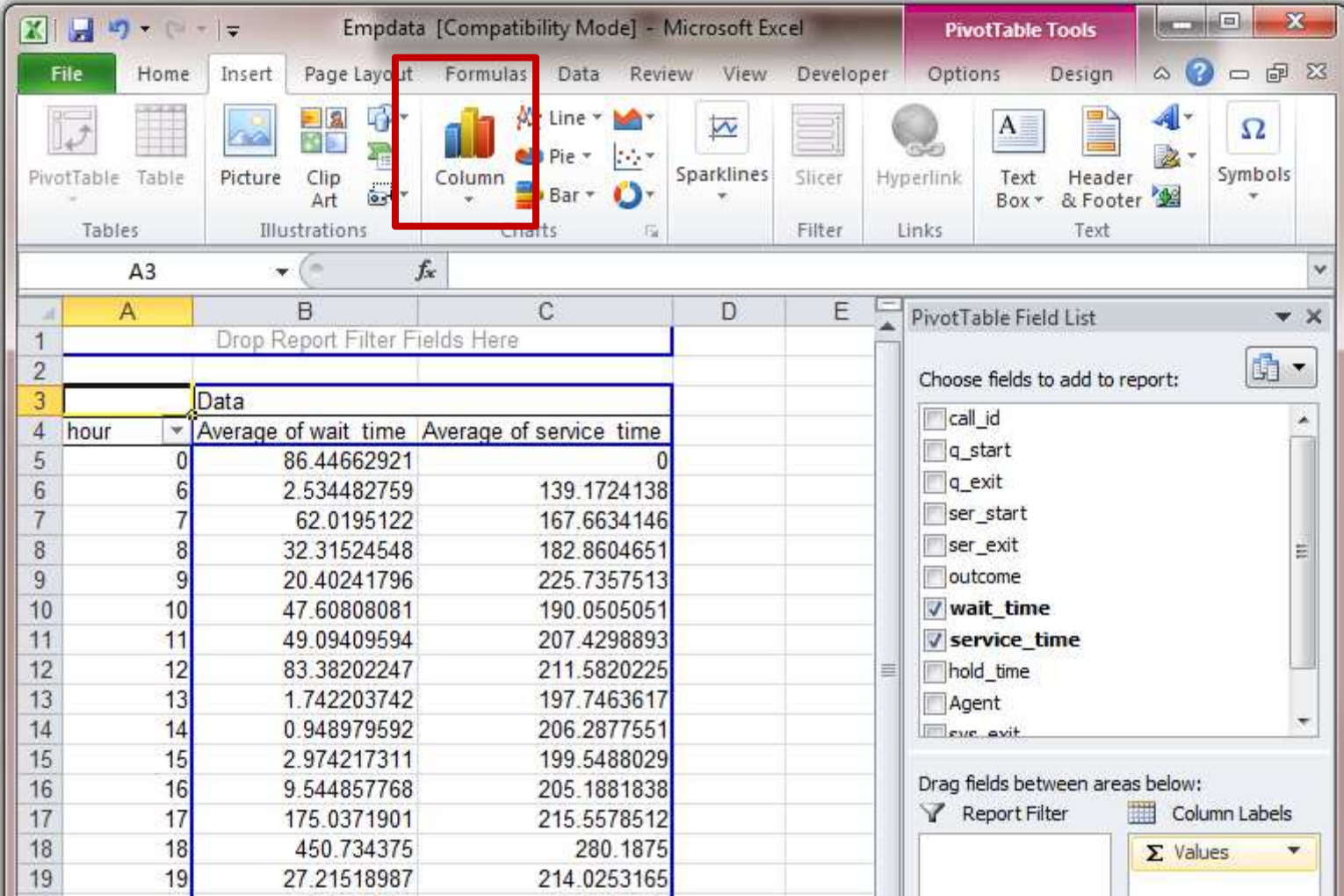
The screenshot displays the Microsoft Excel interface with a PivotTable and its field list. The PivotTable is located in the range A3:E4 and shows the average wait and service times for different hours. The PivotTable Field List on the right shows the fields available for the report, including call\_id, q\_start, q\_exit, ser\_start, ser\_exit, outcome, wait\_time, service\_time, hold\_time, Agent, and exe\_exit. The wait\_time and service\_time fields are selected for the report. The Report Filter section shows the hour field, and the Column Labels section shows the average of wait and service times. The Row Labels section is empty.

**PivotTable Data:**

| hour | Average of wait time | Average of service time |
|------|----------------------|-------------------------|
| 0    | 86.44662921          | 0                       |
| 1    | 2.534482759          | 139.1724138             |
| 2    | 62.0195122           | 167.6634146             |
| 3    | 32.31524548          | 182.8604651             |
| 4    | 20.40241796          | 225.7357513             |
| 5    | 47.60808081          | 190.0505051             |
| 6    | 49.09409594          | 207.4298893             |
| 7    | 83.38202247          | 211.5820225             |
| 8    | 1.742203742          | 197.7463617             |
| 9    | 0.948979592          | 206.2877551             |
| 10   | 2.974217311          | 199.5488029             |
| 11   | 9.544857768          | 205.1881838             |
| 12   | 175.0371901          | 215.5578512             |
| 13   | 450.734375           | 280.1875                |
| 14   | 27.21518987          | 214.0253165             |
| 15   | 1.575                | 215.3375                |
| 16   | 0.724137931          | 169.9655172             |
| 17   | 0.977777778          | 229.8222222             |
| 18   | 42.11255797          | 190.3892258             |

# Using Pivot Table (6)

- Drawing graphs using Pivot Table



The screenshot shows the Microsoft Excel interface with the 'Empdata [Compatibility Mode] - Microsoft Excel' window. The 'Insert' tab is active, and the 'Charts' group is highlighted with a red box. The 'Column' chart icon is selected. The PivotTable is located in the range A3:E19. The PivotTable Field List task pane is open on the right, showing the 'wait\_time' and 'service\_time' fields selected for the report.

| hour | Average of wait time | Average of service time |
|------|----------------------|-------------------------|
| 0    | 86.44662921          | 0                       |
| 6    | 2.534482759          | 139.1724138             |
| 7    | 62.0195122           | 167.6634146             |
| 8    | 32.31524548          | 182.8604651             |
| 9    | 20.40241796          | 225.7357513             |
| 10   | 47.60808081          | 190.0505051             |
| 11   | 49.09409594          | 207.4298893             |
| 12   | 83.38202247          | 211.5820225             |
| 13   | 1.742203742          | 197.7463617             |
| 14   | 0.948979592          | 206.2877551             |
| 15   | 2.974217311          | 199.5488029             |
| 16   | 9.544857768          | 205.1881838             |
| 17   | 175.0371901          | 215.5578512             |
| 18   | 450.734375           | 280.1875                |
| 19   | 27.21518987          | 214.0253165             |

# Using Pivot Table (7)

- Drawing graphs using Pivot Table

