# Little's Law and Capacity Analysis

# Case 1. Workforce Management

- A Japanese company has 36,000 employees, 20% of whom are women. The average term of employment for a woman is 37 months, whereas for men it is 200 months. (Assume that the total employment level and the mix of men and women are relatively stable over time.)

- **Q1**: How many new employees per year does the company hire?
- **Q2**: What is the average term of employment for all employees?

# Case 1 - Q1

- **Q1**: How many new employees per year?
- For women,
  - $L_W$ = average number of women in system
    = 36, 000 × 0.2 = 7, 200 women
  - $W_W$ = 37 months
  - $\lambda_W = \frac{7200\ women}{37\ months} = 194.6$ women/month
    = average number of new women employees hired per month
- Similarly, for men,
  - $L_M$ = 36, 000 × 0.8 = 28,800 men and $W_M$ = 200 months
  - $\lambda_M = \frac{28800\ men}{200\ months} = 144$ men/month

→ 194.6 + 144 = 338.6 new employees per month,

→ equivalently, 338.6 × 12 = 4063.2 new employees per year

# Case 1 – Q2

- **Q2**: average term of employment for all employees?
- W=0.2 x 37m + 0.8 x 200 m = 167.4 m? WRONG!
- $W = \frac{L}{\lambda} = \frac{36000 \text{ employees}}{338.6 \text{ employees per m (Q1)}} = \mathbf{106.3\ m}$
- Then, what are the right proportions for calculating W?
  - $W = \frac{L}{\lambda} = \frac{L_W + L_M}{\lambda_W + \lambda_M} = \frac{\lambda_W W_W + \lambda_M W_M}{\lambda_W + \lambda_M} = \frac{\lambda_W}{\lambda_W + \lambda_M} W_W + \frac{\lambda_M}{\lambda_W + \lambda_M} W_M$
  - Note $\frac{\lambda_W}{\lambda_W + \lambda_M} = 0.57 \neq \frac{L_W}{L} = 0.2$ and $\frac{\lambda_M}{\lambda_W + \lambda_M} = 0.43 \neq \frac{L_M}{L} = 0.8$

# Case 2. Improving a Process

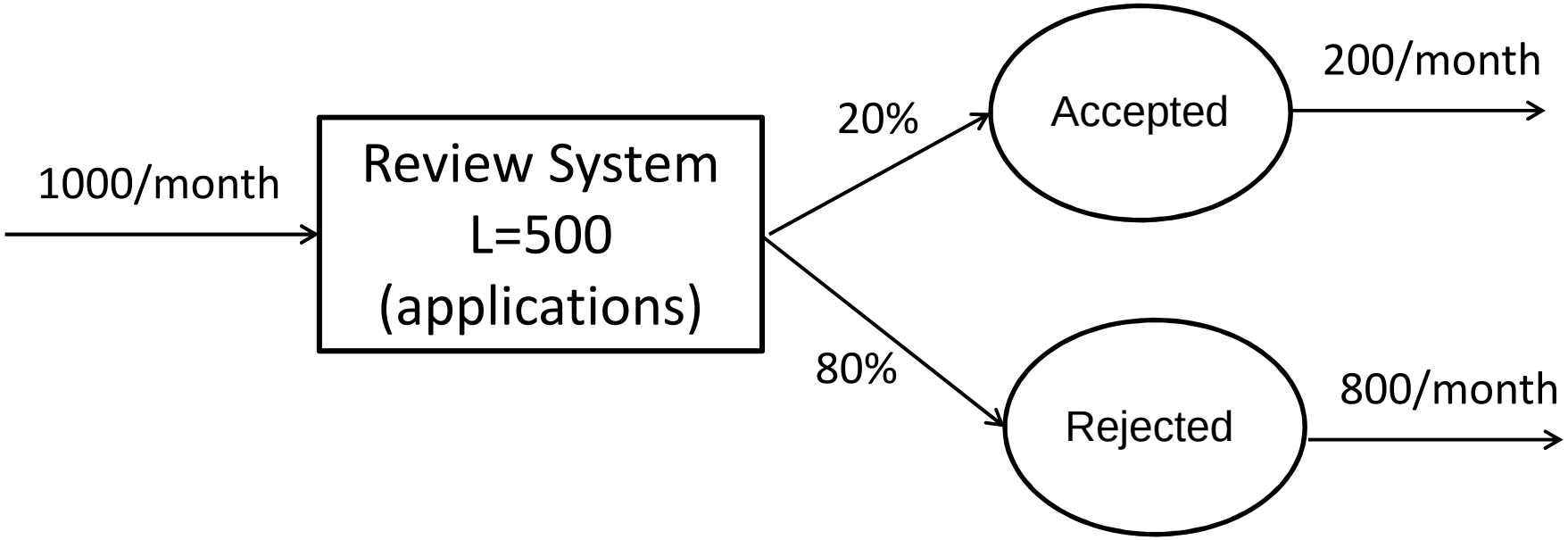
- MBPF Finance Inc. makes loans to qualified buyers of prefabricated garages. It receives about 1000 loan applications per 30-day working month and makes accept/reject decisions based on an extensive review of each application.

- Prior to January 1998 (under "**Process 1**") MBPF Finance processed each application individually. On average, 20% of all applications received approval. An Internal audit showed that, on average, MBPF Finance had about 500 applications in process at various stages of the approval procedure, but on which no decisions had yet been made.

- **Q1**: What was the **average processing time** for a request under Process 1?

# Case 2. Q1

- **Q1**: **average processing time** for a request?




$$\rightarrow W = \frac{L}{\lambda} = \frac{500 \text{ applications}}{1000 \text{ applications per month}} = 0.5 \text{ month} = \mathbf{15} \text{ days}$$

# Case 2. Improving a Process (2)

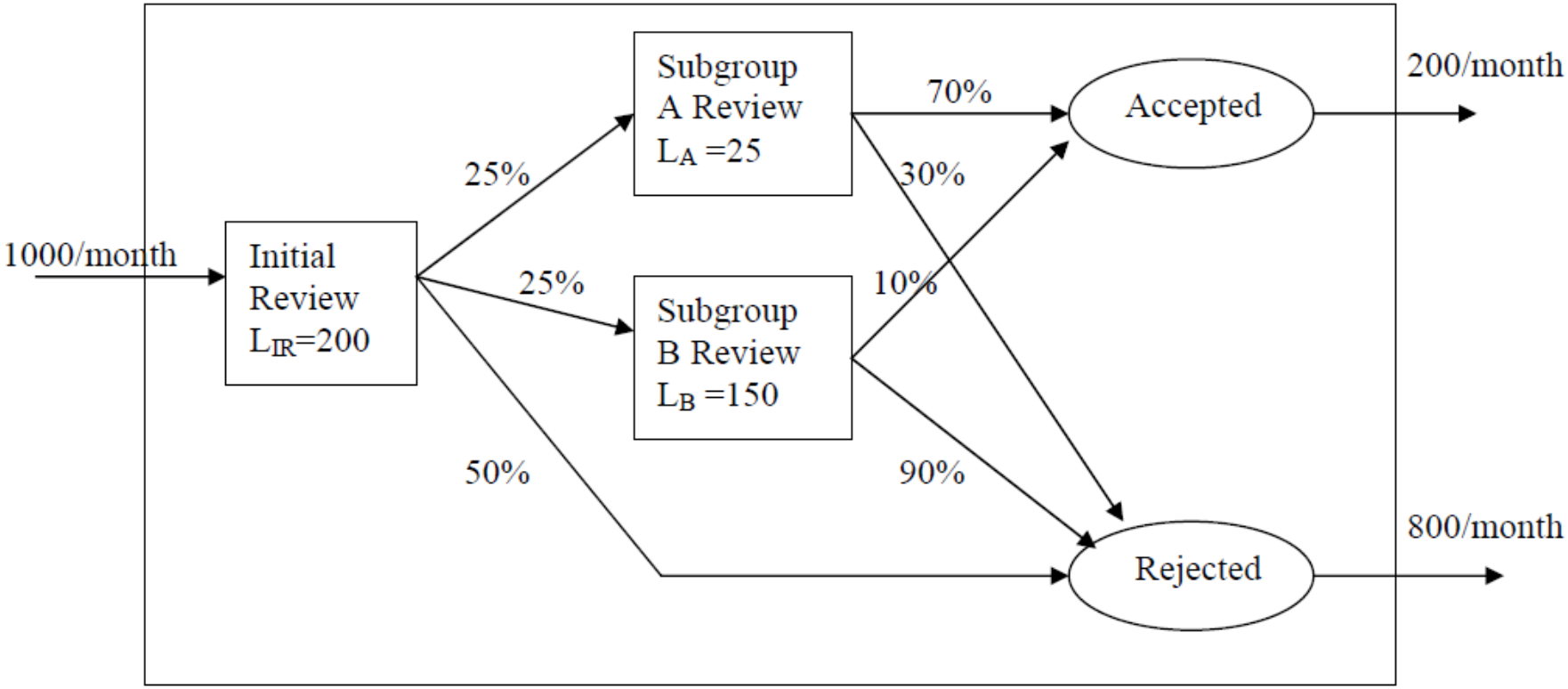
- In response to complaints on long processing time, MBPF hired a consulting company that suggested the following changes, creating "**Process 2**".
  1. Low percentage of approved applications: An initial Review Team should be set up to *preprocess* all applications according to strict but fairly mechanical guidelines.
  2. Each application would fall into :

     (A)looks excellent

     (B)needs more detailed evaluation

     (C)reject.
  3. Type A and B would be forwarded to different specialist subgroups. Each subgroup would then evaluate the applications in its domain and make accept / reject decisions.
- Process 2 was implemented on an **experimental** basis. The company found, on average, 25% of all applications were of type A, 25% were of B and 50% were of C. Typically, about 70% of type A and 10% of type B were approved on review. On average, 200 applications were with the Initial Review Team undergoing pre-Processing. Only 25 were with the subgroup A Team undergoing the next stage of Processing and approximately 150 were with the subgroup B Team.

# Case 2. Improving a Process (3)



# Case 2. Q2

- **Q2**: What is the average processing time of a request under Process 2?

1. Treating it as one big processing station:
   - L = average requests undergoing processing = $L_{IR}+L_A+L_B$ = 375

     W = 375/(1000 per month) = **11.25** days (**reduced by almost 4 days!**)

2. Looking at different "Processing stations":
   - W (IR) = 200/ (1000 per month) = 0.2 month = **6** days
   - W (A) = 25/ (1000*0.25 per month) =0.1 month = **3** days
   - W (B) = 150/ (1000*0.25 per month) =0.6 month = **18** days
   - → W = 0.5 x **6** days + 0.25 x (**6** days + **3** days) + 0.25 x (**6** days + **18** days)

     = **11.25** days

# Case 2. Q3

- **Q3**: Did the processing time for approved requests decrease from Process 1 to Process 2 ?

- Process 1: all applications spend **15** days

- Process 2:
  - $W^{Approved}$ = W(IR) + W(A or B) for accepted applications

    $= 0.2 \text{ month} + \frac{(L_A \text{x } 0.7 + L_B \text{ x } 0.1)}{200 \text{ per month}} =$ **10.875** days

  → Processing time for approved requests **decreased**

# Case 2. Q3- continued

- **Q3-continued**: Can we say the service level for approved requests improved from Process 1 to Process 2?

- Depends on how we want to measure service level
  - For ALL approved requests, the service level improved

- However, averages do not tell the whole story. Approved requests going through B "enjoy" a 24 day processing time
  - $W_B^{Approved}$ = W(IR) + W(B) for accepted applications
    = 0.2 m + ($L_B$ x 0.1) / (1000 x 0.25 x 0.1 per m) = **24** days
  - Only 1/8 ($= \frac{25\% \text{ x } 10\%}{20\%}$) of the approved requests go through B, so maybe it's worth it?

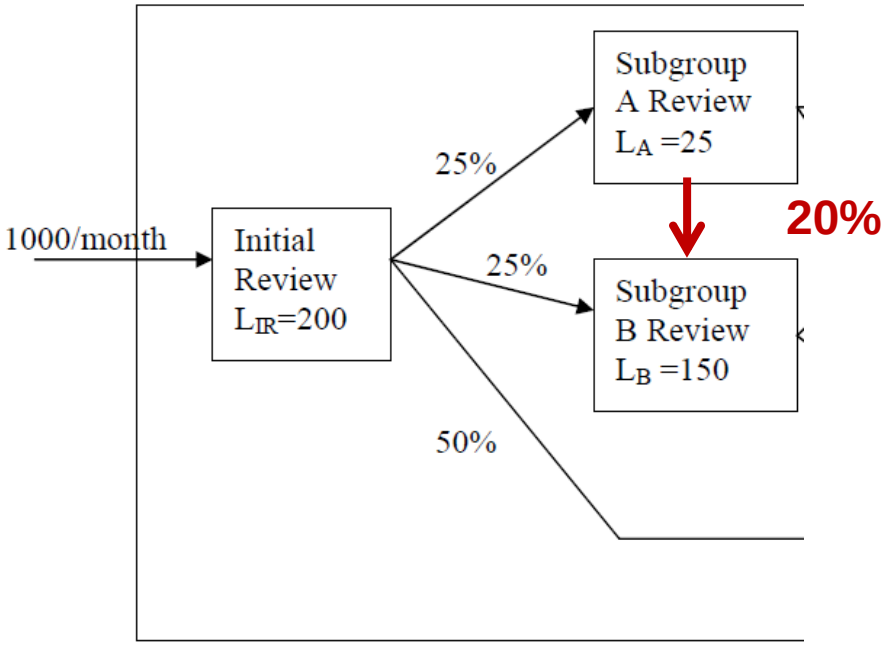
# Case 2. Q4

- **Q4**: Assume 5 Insurance Agents are working in subgroup A (and they share the load evenly). Each agent can handle, on average, 75 requests per month. What is the average utilization level of the agents in subgroup A? Similarly, assume there are 8 agents in subgroup B, each capable of handling 40 requests per month. What is the average utilization level of the agents in subgroup B?

- Subgroup A:
  - Utilization = input rate to A / processing capacity
    = 250 / (5 * 75) = 2/3
- Subgroup B:
  - Utilization = input rate to B / processing capacity
    = 250 / (8* 40) = 0.78125

# Case 2. Q5

- **Q5**: Assume now that, on average, 20% of the requests sent initially to subgroup A need in fact more detailed processing. These requests are sent to B after they complete the processing at A. What is now the utilization level of B agents?




- Utilization = input rate to B from (Initial review + from A) / processing cap

  = (250 **+ 50**) / (8* 40) = 0.9375

# Case 2. Q6

- **Q6**: What is the utilization profile of agents in B, That is, what are the fractions of time, in the long run, that they allocate to requests from Initial Review, to requests from A, and what is the fraction of idle time?

- 250/320=78% of the time is dedicated to requests from Initial Review (Q4)
- 50/320=16% of the time is dedicated to requests coming from A (Q4, Q5)
- 20/320=6% of the time agents are idle

# Homework 2

- 12 questions related to Little's Law and Capacity Analysis
  - Solutions for Questions 3,4,5,6,7,9 on the course website.
  - Submit questions 1,2,8,10,11.
    - Questions 10 and 11 are continuation of Question 9.