

## Class 11

### **A Single-Server Service-Station in Steady State; Multi-Server Service-Stations in Steady State; Laws of Congestion.**

#### **A Non-Parametric Model of A Single-Server Service-Station**

- Analytical models (vs. Simulation/4CallCenters):

“Approximate” analysis of Exact models – Today;

vs. “Exact” analysis of Approximate models – Birth & Death Queues, most notably Erlang-A/C/B (as well as Fluid Models).

- A Non-Parametric Model: the GI/GI/1 Queue.

Lindley’s Equations; Stability.

Tentative: MOP’s; Brummelle’s Formula.

Khinchine-Pollaczek Formula (with an illuminating proof: Hall, pages 168-169).

Allen-Cunneen Approximation (for averages: (5.69) on page 153 in Hall).

Kingman’s Exponential Law of Congestion.

Approximations (Framework for).

Tentative: Priorities: Non-Preemptive, Preemptive.

Tentative: On Optimal Scheduling: The  $c\mu$ -rule. ]

#### **Models of a Multi-Server Service-Station:**

#### **Non-Parametric (GI/GI/m) and Markovian (M/M/m)**

- Congestion Curves
- From M/M/m to G/G/m; (Laws of congestion: Kingman, Allen-Cunneen)
- Strategic Queueing Theory
  - Economies of Scale (EOS) Simply Cases, more Subtle Cases, City Bank
  - Efficiency-Driven Service Operations
  - Pooling in a Queueing Network - Part I
    - Pooling Servers(Capacity): One Fast vs. Several Slow
    - Pooling Queues (Geography): Virtual Call Centers
    - Pooling Tasks (Services): Job Design (Perhaps Later)
  - Kleinrock’s Cycle: Scale-Up (Pooling Queues), then Technological Improvement (Pooling Servers)
- Tentative: Introduction to QED Services Operations

#### **Laws of Congestion**

#### **Recitation 12: MJP Models of Service.**