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TRAFFIC DELAYS AT TOLL BOOTHS

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The collection of vehicular tolls at Port Authority tunnels and bridges is one of the most important operations conducted by the police personnel. More than 250 traffic officers are utilized, and the payroll costs exceed a million dollars annually. In staffing its toll plazas the Port Authority attempts to handle traffic with a minimum number of toll collectors consistent with uniformly good service to the public and properly spaced relief periods for the toll collectors. This requires finding the level of traffic delays that gives the best compromise between the conflicting objectives of economy and service. In the past the number of toll collectors provided for operating a toll plaza was determined by judgment based on experience and a rule-of-thumb work standard which had not been related to service. Judgment was likewise used to allocate manpower and control the number of toll booths opened at any time. This method resulted in patron delays observed to vary from 2 to 50 sec. The tools of probability theory provide methods for dealing with the problem in quantitative terms. They enable determination of the relations between traffic volumes, number of toll booths, and grade of service. With this knowledge the optimum grade of service can be established in a logical manner and the number of toll booths required at any time of day can be specified in advance. Use of this method permitted savings in toll collection expenses and better service.

THE BUSINESS OF THE PORT OF NEW YORK AUTHORITY is public service, which it renders by the construction and operation of various facilities and the promotion and protection of commerce in the Port district. Its operations involve a variety of things, including at airports such items as ramp coordination, fire-fighting and other emergency work, baggage handling, and parking-lot operations; at seaports such items as dock-space allocation, warehousing, and materials handling; at land terminals such items as truck loading, bus loading and dispatching, and rail and truck freight distribution; at tunnels and bridges such items as vehicular-traffic control, accident prevention, and the collection of vehicular tolls, with which this paper deals. Although the list is incomplete, I believe it is sufficient to indicate a fertile area for operations research.

Operations-research methods are being applied to this public service by the Operations Standards Division of the Operations Department, a staff department filling the role of consultant on operating problems encountered by four line departments, each of which is responsible for the physical and financial results of one of the four groups of facilities previously mentioned. OR methods are now being extended to the Comptroller's Department, where sample auditing of various accounts is being investigated. The division's introduction to operations research came about during a comprehensive study of the Port Authority police force—a group of 1000 men comprising the largest single class of employees in the Port Authority.

The purpose of the police study was to determine whether the police staffing of the various facilities was sound and economical. Achieving this purpose necessitated careful analysis of the numerous operations conducted by the police and the establishment of standards for these operations. Good standards are sometimes rather difficult to establish, and the complete police study, which was originally scheduled to take 6 months, actually required 14. The additional time was largely consumed in operational analyses—such as the one covered in this paper—which were not foreseen in the beginning, but which proved to be well worth while. The annual operating savings effected soon after completion of the study amounted to more than ten times the cost of the study itself with potential future savings of more than twenty times the study cost. These are annual savings, repeated each year. In addition, capital savings were achieved of nearly ten times the study cost. Operations research can be credited with important portions of these financial results, and for such other results as better service to the public and benefits to police personnel.

TOLL COLLECTION

The collection of vehicular tolls is a major part of the Port Authority police operations—more than one-fourth of the police personnel are utilized in this function. In the preliminary stages of our analysis, it was observed that the results obtained from toll operations were not altogether satisfactory. The quality of the service varied appreciably from time to time, being considerably better than necessary in some instances, thus involving idle toll collectors; and being unsatisfactory in others, resulting in patron complaints. The average delay, for instance, was observed to vary from 2 to 50 sec.

Prior to our operational analysis, toll booths were manned almost entirely on the basis of opinion and judgment and the manpower supplied was first determined by budget procedures. A facility included in its budget the number of toll collectors it believed was required in the forthcoming year. These requirements were then reviewed by management in the light of the expected annual traffic, past experience, and a rule of thumb about how much traffic could be handled by a toll collector. The manpower authorized and provided by this budget procedure was then allocated by the facility to various days of the week and tours, and was based on the composite judgment of the toll sergeants, who supervised the toll operations, and their superiors. This is a typical management process.

On a given tour the actual number of toll booths manned at any particular time was left to the discretion of the toll sergeant on duty, who made the best use he could of the manpower at his disposal. Toll sergeants are rotated around the three tours and alternated between tolls and traffic duty, making it difficult for a sergeant to become thoroughly familiar with traffic on any tour. The principal operation required of the toll sergeant is compromising the frequently conflicting requirements of traffic on the one hand with personal and meal reliefs for the toll collectors on the other. Since the toll sergeants have varying experience and different ideas about how to operate, the results were not consistent. Some exercised good judgment, and some did not; interviews with toll collectors indicated that their relief requirements were in too many cases being unsatisfactorily met. Precedence was generally given to the patron at the expense of the toll collector when conflicts arose, but, since toll collecting is a rather nerve-wracking job, extended working periods without a relief are very undesirable.

From the foregoing discussion, the general objectives of the study can be seen to be (1) to evaluate the grade of service given to patrons and determine how it varies with the volume of traffic handled by toll lanes; (2) to establish the optimum standards of service; and (3) to develop a more precise method of controlling expenses and service while at the same time providing for well-spaced reliefs to the toll collector.

OBSERVATIONS

The first type of data recorded was traffic arrivals at the toll plaza. One observer counted the number of vehicles arriving in 30-sec intervals and recorded the count along with the time, as shown in the first two columns of Table I. Intervals of 30 sec were found to be about the shortest that could be used to permit the observer to make recordings without losing the count.

The second type of data recorded was the extent of the backup in each open toll lane. These data, recorded by a second observer, were also taken at 30-sec intervals and in synchronism with the traffic arrival recordings.

The third type of data was the toll transaction count. These data were recorded at half-hourly intervals and whenever there was a change in the number or type of toll lanes. In some cases the number and type of lanes opened were left to the toll sergeant, but in other cases the number and type were regulated by the survey group in order to obtain information on specific arrangements and to create moderate amounts of congestion. These data provide a check on the arrival count, with which they should agree when adjusted for the change in accumulation at the beginning and end of

TABLE I

SAMPLE OF RECORDED DATA

Time, P.M.	Traffic arrivals	Vehicles in			Total	Lanes occupied
		Lane A	Lane 6	Lane 10		
8:58						
...	10	2	2	1	5	3
8:59	6	0	1	0	1	1
...	1	0	1	0	1	1
9:00	3	1	0	0	1	1
...	4	0	1	1	2	2
9:01	5	1	1	0	2	2
9:15[a]	6	0	1	0	1	1
...	5	1	2	0	3	2
9:16	6	5	0	1	6	2
...	4	2	1	0	3	2
9:17	4	1	0	1	2	2
...	2	0	0	0	0	0
9:18	7	1	1	3	5	3
Totals[a]............	205	41	55	38	134	76
Transactions[b]						
9:18	...	2102	79785	97466	...	...
8:58	...	2034	79698	97416	...	...
Totals.............	...	68	87	50	205	...

[a] Fourteen minutes omitted.
[b] Similar to cash-register tally number.

an observation period. More importantly, they also permit computations to be made for each lane individually, as well as for all lanes collectively.

Table I shows a sample of all three types of data, taken at the Lincoln Tunnel when three left-hand toll booths were open in one direction and were handling traffic at the rate of 615 vehicles per hour. Table I also shows the preliminary steps taken in analyzing the date, these being the totals of each column, the total backup for the three lanes at each observation, and the number of lanes occupied at each observation.

COMPUTATIONS

One of the principal factors of interest is average delay. It is first desirable, however, to calculate the over-all time taken per vehicle to clear the toll lanes; this includes both the delay, or waiting time, and the booth holding or servicing time. The over-all time used by all vehicles to get through the toll lanes, based on the sample observations, is 4020 sec, and the average is 19.6 sec.

The total booth time used in handling vehicles during the observation period of Table I is the total number of occupancies observed—given by the total of the last column—multiplied by the observation interval, or 2280 sec. The average booth holding time is 11.1 sec. The average delay per vehicle is the over-all time per vehicle less the booth holding time, or 8.5 sec.

Another item of interest is the average delay expressed as a multiple of holding time, which I shall call 'delay ratio.' This item is of particular interest because of its use in delay theories, and also because it provides a measurement of delay that is independent of fluctuations in holding time. The delay ratio is average delay per vehicle divided by average booth holding time, or 0.77.

The percentage of vehicles delayed might well be used as a measurement of the grade of toll-booth service. This can be obtained by counting the number of instances in which two or more vehicles were observed at a single booth and dividing this count by the total number of booth occupancies observed. Another factor is average delay to delayed vehicles. This is the average delay to all vehicles divided by the percentage delayed.

The maximum delay can be estimated from the maximum backup and the average booth holding time. In the example, the maximum backup observed was six vehicles. This is found by inspection of the data. The sixth vehicle waited for the five preceding ones, each assumed to have taken the average booth holding time. Thus, the maximum delay is 55.5 sec.

The availability of an empty toll lane is still another factor that could be used to measure the grade of toll-booth service. At first thought one might state that this is complementary to the percentage of vehicles delayed, since any vehicle may go either into an occupied lane and be delayed or an empty lane and not be delayed, and if drivers always picked an unoccupied lane when available, this would be the case. Unfortunately, however, drivers often pick an occupied lane instead of an empty one even though an empty lane is always available, and in so doing can delay all vehicles. The number of instances when there were one or more lanes empty in the example was 31 out of 40, thus giving a percentage availability of 77.5. The complement to the percentage delayed is 55.

In addition to the previously mentioned items, any one of which could be used to specify the grade of toll-booth service being given, there is interest in the percentage occupancy of the toll booths, which is given by the number of occupancies observed, divided by the total number of observations. In the example, this is 63.3.

These calculations have been made for the three toll lanes as a group. By using the transaction counts shown at the bottom of Table I, all the items can be calculated for each lane individually.

Having shown how a number of tentative service criteria could be determined from the data, we shall, in the balance of the paper, concern ourselves with only those that were actually used to arrive at service standards. Before going into an analysis of these, let us consider the analysis of traffic arrivals.

TRAFFIC-ARRIVAL ANALYSIS

The traffic-arrival patterns were analyzed by forming frequency distributions of the number of vehicles arriving in 30-sec intervals at various volumes. Observations were formed into 200 vehicles-per-hour groups, and in each group the number of occurrences of arrivals of 0, 1, 2, 3, etc. vehicles were counted and organized into a table. The empirical frequency of occurrences of each arrival class was computed as a percentage of the total number of intervals observed. These percentages were then plotted against the arrival classes, as shown in Fig. 1, and frequency polygons were drawn. These frequency distributions have rather good resemblances to the distributions one would expect with pure-chance traffic. One feature to be noted, however, is the tendency for the right-hand tails of the distributions at the higher volumes to be somewhat prolonged. The extension of the distribution for the highest volume shown out to 28 along the abscissa should be noted in particular.

Comparison for the same volumes of traffic can be made with the theoretical distributions which are shown in Fig. 2. The similarity to the actual distributions is quite evident; however, it will be noted that the right-hand tails are not as prolonged. These theoretical distributions are Poisson at the lower volumes and normal at the higher volumes.

A more easily observed comparison between the actual and the two theoretical distributions is shown in Fig. 3, where they are plotted together. These distributions pertain to a volume of 655 vehicles per hour at the Lincoln Tunnel. The mean arrival rate is 5.46 vehicles per 30-sec interval, and the standard deviation is 2.73 vehicles per 30-sec interval. In computing the Poisson and normal values the sample mean was used, but in the case of the normal distribution the standard deviation used was the theoret-

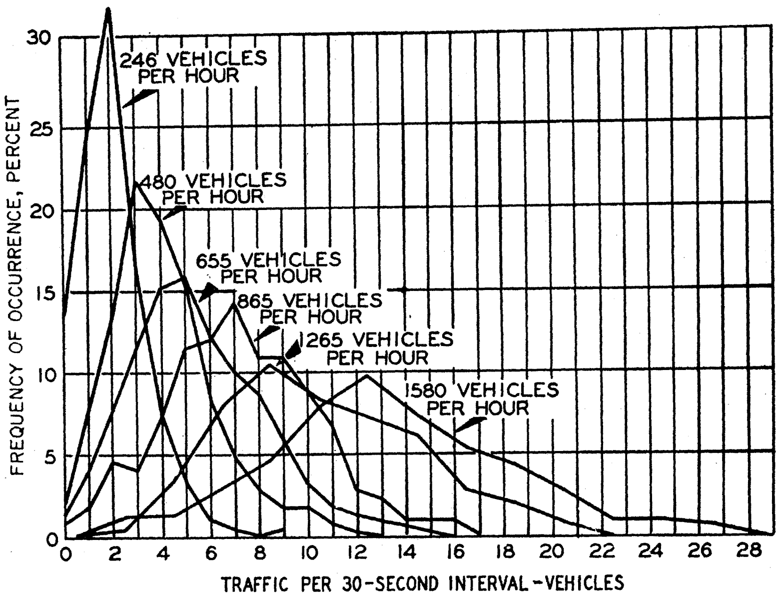

FIG. 1. Frequency distribution of traffic arrivals.

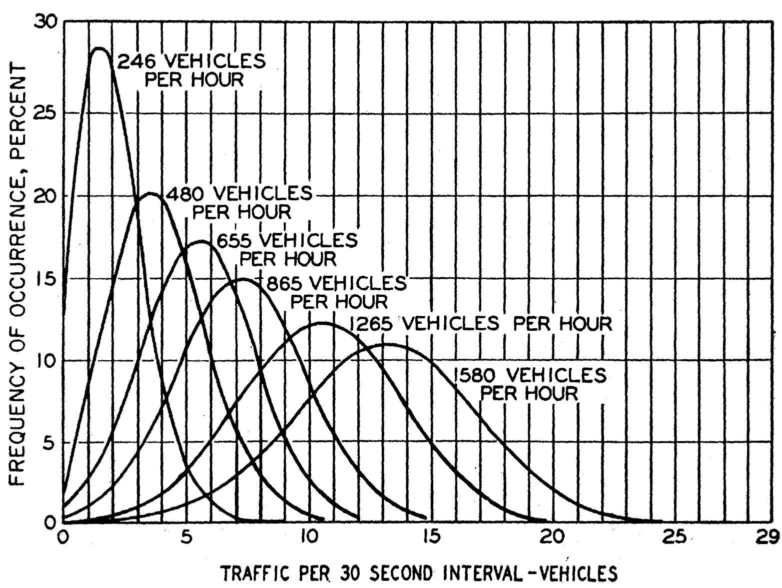

FIG. 2. Theoretical frequency distribution of traffic arrivals.

ical one for a pure-chance distribution. In this example the Poisson, shown solid, appears to give a better fit to the actual than the normal.

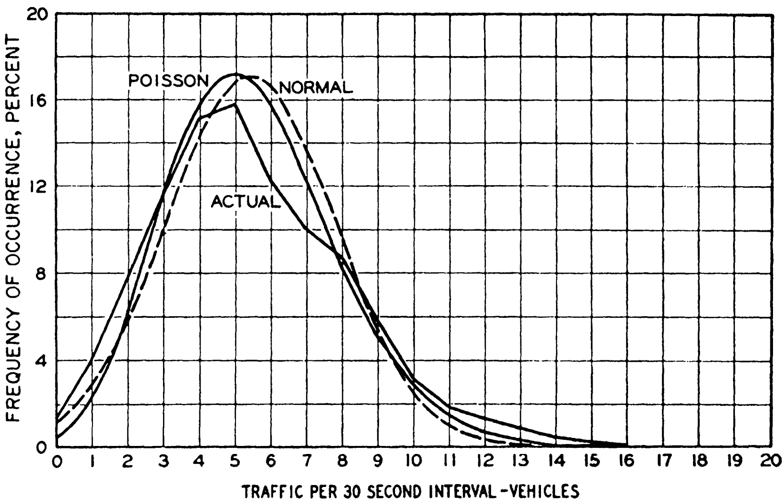

FIG. 3. Comparison of actual and theoretical traffic arrivals for 655 vehicles per hour at Lincoln Tunnel.

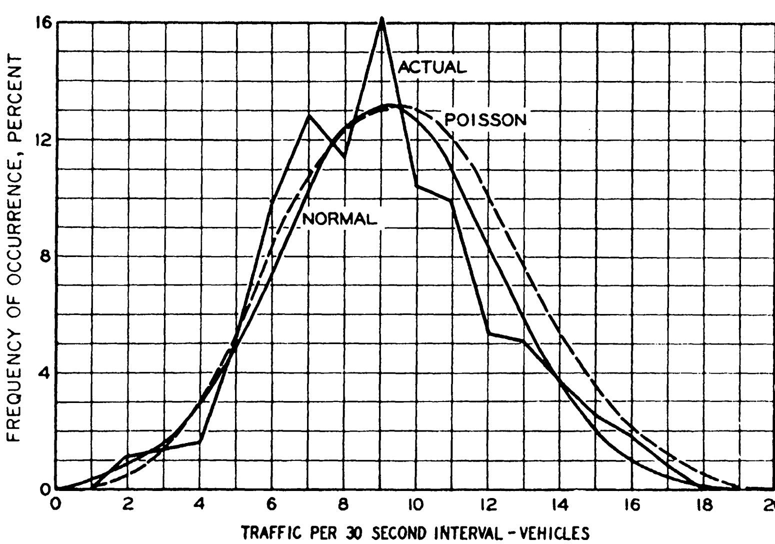

FIG. 4. Comparison of actual and theoretical traffic arrivals for 1100 vehicles per hour at George Washington Bridge.

The arrival distributions at the George Washington Bridge, as well as at the Lincoln Tunnel, were analyzed in the same manner with similar results, as shown in Fig. 4. This figure applies to a volume of 1100 vehicles per hour, with a mean of 9.17 and a standard deviation of 3.00. Here the normal curve, shown solid, appears to fit slightly better than the Poisson.*

Table II shows the goodness of fit for a number of traffic volumes investigated. There is a very evident tendency for the fit of both distributions to deteriorate at the higher traffic volumes, although both continue to show a satisfactory fit better than 0.05. This deterioration, however, is of some interest since it corresponds with the extended right-hand tails of the actual distributions that were previously noted. Both the extended tails and the poorer fit at higher volumes can be explained by the development of congestion, which causes the operation of one vehicle to interfere with the operation of another. At still higher volumes it is apparent that the fit would break down, and under bumper-to-bumper congestion the distribution would tend to become constant. The volume at which the fit becomes unsatisfactory depends, of course, on the number of lanes in the roadway. The column indicating the theoretical better fit is based on the theory that the Poisson expresses the better approximation to the binomial below a mean value of 5 and the normal the better approximation above this value. This mean corresponds with a traffic volume of 120×5 equals 600 vehicles per hour. Two exceptions to this theory, noted by asterisks in the table,

* The Poisson distribution is given by the expression $P(x)=e^{-m}m^x/x!$, where $P(x)$ in this case is the probability of x vehicles arriving in any interval when the average arrival rate is m. It will be noted that the Poisson distribution is fully specified by a single parameter, the mean.

The normal distribution is given by the expression

$$F(x)=\exp[-(x-m)^2/2s^2]/s\sqrt{2\pi},$$

where $F(x)$ is the probability of x vehicles arriving in any interval when the average arrival rate is m and the standard deviation is s. For pure-chance traffic, where p is the probability of any random vehicle arriving in a particular interval, q the probability of it not arriving, and n the total number of vehicles in the hour, the standard deviation is $s=\sqrt{npq}$.

Both of these distributions are close approximations of the binomial distribution given by the expression $P(x)=C_x{}^n p^x q^{n-x}$, when n, the number of vehicles, is large and p is small. In the distribution of hourly vehicular traffic arrivals in 30-sec intervals, small p is 1/120, q is 119/120, and n is the total traffic volume. However, our interest is more in the Poisson and normal distributions than the binomial, since they are easier to deal with.

To learn which of these two theoretical distributions gives the better fit, the chi-square test of fitness can be used. The chi-square value is given by the expression

$$\chi^2=\sum_{x=0}^{n}(f_o-f_t)^2/f_t$$

where f_o is the observed frequency and f_t the theoretical. When these values have been computed, they may be looked up in a table of chi-square values to obtain the probability level of fit. A perfect fit would show a probability level of 1.00, but a fit showing a probability level better than 0.05 is generally taken as satisfactory.

are not considered significant. The results support the belief that the true distribution, before congestions enter as a factor, is binomial, and consequently is a pure-chance distribution.

OCCUPANCY VS DELAY RATIO

Having established the randomness of traffic we thought that we would be able to draw curves of traffic volumes vs each of the various service criteria, and then find a delay theory that would agree closely enough with the empirical curves for at least some of these criteria to be predictable from theory. Unfortunately, such was not the case; for some delay factors satisfactory empirical curves could not be drawn because of the wide dispersion of points. To determine accurately the correlation curves for some of the service criteria directly from computed points would have required a very large amount of data.

TABLE II

TRAFFIC-ARRIVAL GOODNESS OF FIT

Traffic volume	Poisson	Normal	Theoretical best fit
246	0.754	0.235	Poisson
480	0.966	0.743	Poisson
655	0.738*	0.459	Normal
865	0.842	0.882	Normal
1100	0.718	0.812	Normal
1265	0.359*	0.295	Normal
1580	0.191	0.575	Normal

The most obvious relation to seek to establish—that between traffic volumes and average delay in sec—fell into this category. One reason for this is that average delay measured in sec is a function not only of traffic volume but also of booth holding time. Because of differences in traffic composition, holding times are different at different facilities, and the data taken at one facility are not usable for another. Another factor is that holding time is partly under the control of the toll collectors, who in some cases knew they were being observed and were naturally influenced to keep holding times lower than usual. These factors made the direct plotting of average delay for each facility unsatisfactory.

To get around the difficulty our attention was directed to curves of occupancy vs delay ratio. This relation is independent of holding time, permitting data from different facilities to be combined. The scattering of the points was appreciably reduced and, with the greater number of observations available from combining all facilities, satisfactory empirical curves could be drawn. A further consideration is that delay theory is developed on the basis of holding-time units, and it was desired to compare the empirical results with the theories of Erlang, Molina, and the joint theory of Pollaczek and Crommelin.

Erlang's theory is given by the formula

$$d=\frac{[y^x/x!]\,[x/(x-y)^2]}{\{1+y+(y^2/2!)+(y^3/3!)+\cdots+[y^{x-1}/(x-1)!]+(y^x/x!)[x/(x-y)]\}}$$

where d is average delay in units of holding time, x is the number of traffic channels, and y is the traffic intensity in erlangs.[1] An erlang is defined as the average occupancy during a time T, divided by T. It is a dimensionless unit, being similar in this respect to the decibel used to express values of attenuation. For example, if three channels are each occupied one-half the time of a period T, the total occupancy is $1.5T$ and the traffic intensity is 1.5 erlangs. The number of erlangs also expresses the average number of traffic elements handled simultaneously. The delay formula by Erlang is based on an assumption of exponentially distributed holding times, where $P(t)=e^{-t/h}$ gives the probability of an element of traffic selected at random having a holding time of at least t, when the average holding time is h.

Molina's formula constitutes a correction factor applied to Erlang's formula to alter it for constant holding-time distribution.[2] The correction factor is given by the expression

$$[x/(x+1)][1-(y/x)^{x+1}]/[1-(y/x)^x].$$

The Pollaczek-Crommelin formula, based on an assumption of constant holding-time distribution,[3] is given by the expression

$$d=\sum_{w=1}^{\infty}e^{-wy}\left[\sum_{u=wx}^{\infty}\frac{(wy)^u}{u!}-\frac{x}{y}\sum_{u=wx+1}^{\infty}\frac{(wy)^u}{u!}\right].$$

Figure 5 shows a comparison between values predicted by these formulas and the empirical results for a single toll booth. The empirical values are shown as plotted points. It can be seen that, as expected, the Pollaczek-Crommelin formula shows a good fit, whereas both Molina's and Erlang's formulas give delays considerably greater than the empirical results. This indicates that booth holding times are essentially constant in distribution, and that the Pollaczek-Crommelin formula more accurately portrays average delay at the higher occupancies than does Molina's formula, although there is not much choice between them at lower occupancies. A sampling of holding times by means of stop-watch timing also indicated booth holding times were more nearly constant than exponential in distribution.

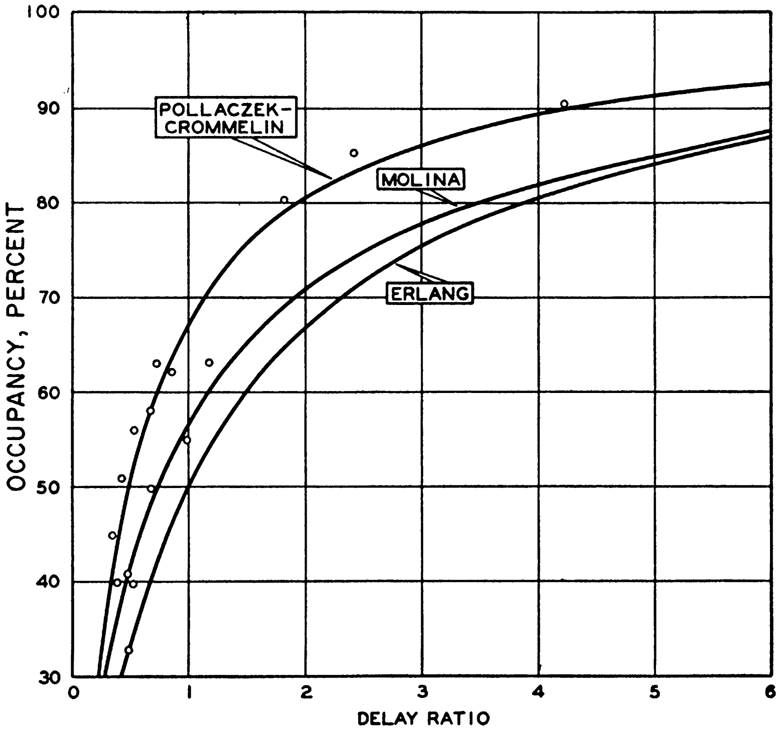

FIG. 5. Comparison of actual points and theoretical occupancy delay curves for one toll booth.

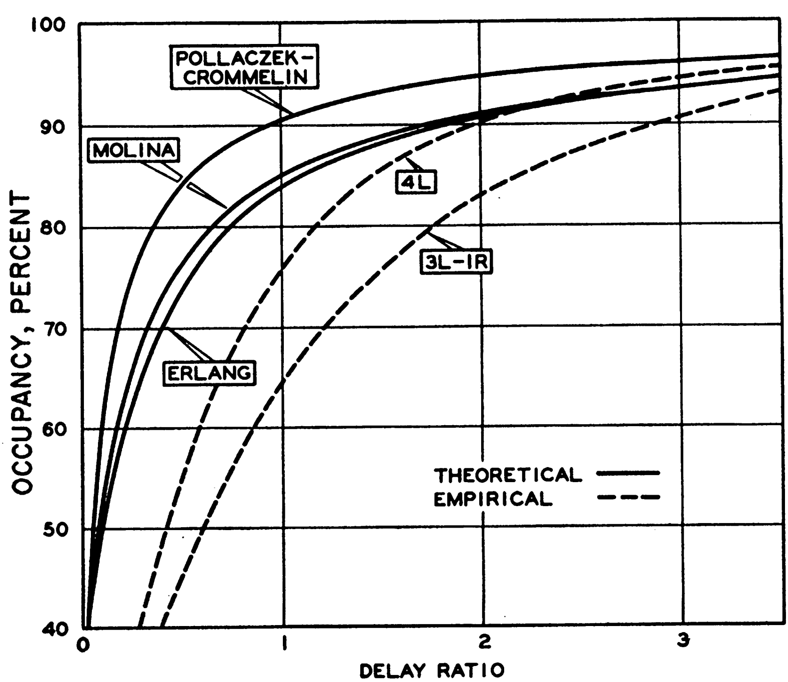

FIG. 6. Comparison of actual and theoretical occupancy delay curves for four toll booths.

In the case of four toll booths, shown in Fig. 6, the empirical results show greater delays than any of the theories, and Erlang's formula for exponentially distributed holding times is closer to the empirical results than the constant holding-time formulas. The reason for this is that previously mentioned—of traffic lining up at one booth while another toll lane is empty. Because the traffic was found to be random, and because of the fit of the Pollaczek-Crommelin formula to one toll booth, this poor traffic distribution is virtually the sole cause of the much greater delays found than that given by the constant holding-time formula. Our efforts to adjust the formula for this factor were not successful, so it was necessary to proceed with empirical values.

It will be noted that two empirical curves are shown for the case of four toll booths, one curve applying to four left-hand toll booths and the other to three lefts and one right. Left-hand toll booths are those on the driver's side of a vehicle passing through the lane, and right-hand toll booths are the opposite. Both curves have been shown to illustrate the inferiority of the right-hand booths.

TABLE III

Comparison of Four Left-Hand with Three Left-, One Right-Hand Booths

Percentage of equal occupancy for 4 left-hand booths	Corresponding percentage for 3L-1R	Delay ratio for 4L	Corresponding delay ratio for 3L-1R	Increase in delay for 3L-1R, %	Value of RH vs LH, %
50	40	0.40	0.60	50	20
60	50	0.60	0.85	41	33
70	60	0.85	1.25	47	43
80	70	1.25	1.80	44	50
90	83	2.00	3.00	50	69

This is illustrated even more clearly in Table III, which shows the percentage increase in delay ratio for equal occupancies and the reduction in occupancy for equal delays when a right-hand booth is substituted for a left. In the first comparison it will be noted that this results in an increase in delay of approximately 50 per cent. The increased delay is suffered by all traffic in the aggregate, not just by the traffic handled at the right-hand booth.

In the second comparison it will be noted that at moderate delay levels a right-hand toll booth has less than one-half the value of a left-hand toll booth. The value of a right-hand booth increases as congestion at the tolls plaza increases, thus indicating the overflow character of the right-hand booths. As a consequence of these findings the Port Authority is reconstructing all major tolls plazas to provide only left-hand toll booths.

TRAFFIC VS HOLDING TIME

To convert the dimensionless ratios of occupancy and delay into the practical units of vehicles per hour and seconds of delay requires a determination of holding-time values. In some delay problems, holding time is unaffected by the traffic congestion and by the number of channels employed. This is the case, for instance, when dealing with telephone traffic. But in the case of toll operations, as shown in Fig. 7, for the George Washington Bridge, holding time was found to be a function of traffic volume and the number of toll booths employed. It can be seen that holding time is appreciably longer at low volumes of traffic per lane than it is at high volumes. As traffic per lane approaches zero, the holding time approaches a maximum value of approximately 13 sec, and as traffic volume rises to

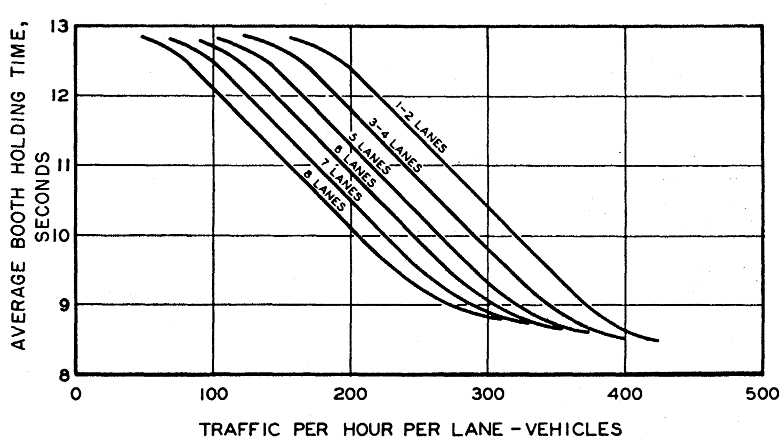

FIG. 7. Average booth holding time per vehicle at George Washington Bridge.

the maximum that can be handled per lane, the holding time approaches a minimum value of $8\frac{1}{2}$ sec. It will also be noted from the figure that the greater the number of toll lanes used, the sooner the holding time begins to drop. However, once it begins to drop, it does so in the same manner for all groups of booths, i.e., in proportion to increases in traffic per lane, the slope being approximately 1 second to 50 vehicles per lane per hour.

The explanations for this phenomenon seem apparent. In the first place, holding time decreases as traffic per lane increases because both toll collectors and patrons tend to expedite the operation under the pressure of backed-up traffic. This seems to be a fairly common phenomenon in waiting problems involving people who are aware of the amount of congestion. In our field observations it was noticed that when traffic was light there was considerably more conversation between collector and patron than when traffic was heavy. Another factor is that when there is a waiting line at a toll booth, patrons have an opportunity, while waiting, to get their tolls ready; whereas with an empty lane, the patron might drive right up to the booth before reaching for this toll—and then have to search to find it.

The explanation for the quicker drop in holding time for larger groups of lanes appears to lie in the nonuniform distribution of traffic between the open lanes. Certain lanes, particularly those having left-hand booths, and those located near the center of the plaza, are considerably favored by patrons over the others. The greater pressure of traffic in these favored lanes brings about a reduction in holding time, even though the average traffic per lane over all lanes may still be low. Since the favored lanes handle the most traffic, they have a proportionately greater effect on the average over-all lanes than do the less favored lanes.

When the curves of traffic per lane vs holding time were plotted, it was found that there were few values at the high traffic volumes to define clearly the location of the curves at these levels of traffic. The reason for this is that to obtain points at heavy loadings per lane required the creation of heavy congestion. This would result in complaints that might be embarrassing to answer. Therefore we sought other methods of finding where the curves leveled off.

The principal method used consisted of stop-watch measurements of toll transaction times and the calculation of vehicle times for various types of transactions. Booth holding time is made up of two parts: One is the time taken by the toll collector to receive the toll from the patron and, if necessary, to give change or a receipt. The other is the time taken by the vehicle to move into toll-paying position. The collection, or transaction, time is taken as the interval between the time the wheels of a vehicle stop rolling when it moves into a lane and the time they again start rolling when the vehicle moves out of the lane. The vehicle time is taken as the interval between the time one vehicle starts to leave and the following vehicle comes to a stop in toll-paying position.

Using this breakdown of the holding time, it was relatively easy to make stop-watch measurements of minimum transaction times, just by watching the wheels of the vehicles as they came to a stop and started up again in lanes having long lines. It would also have been easy to measure the vehicle time in a similar fashion, but this was not considered necessary since information is readily available on the acceleration and deceleration of automotive vehicles, and these times could be determined from available curves.

The observations on transaction times, which were made on several hundred vehicles, and the determination of vehicles times from acceleration-deceleration curves resulted in a breakdown of minimum booth holding times by types of vehicles and types of toll booths. This is shown in Table IV. With this information it is possible to calculate minimum holding times for traffic composed of various percentages of passenger cars, buses, trucks, and tractor-trailer units. For example, traffic at the Lincoln Tunnel is, at peak periods, composed of about 64 per cent passenger vehicles, 15 per cent buses, 14 per cent trucks, and 7 per cent tractor-trailer

TABLE IV

BREAKDOWN OF AVERAGE MINIMUM HOLDING TIMES FOR DIFFERENT VEHICLES

Vehicle	Vehicle time, sec	Toll time, sec		Holding time, sec	
		LH	RH	LH	RH
Passenger car	5.0	3	4	8	9
Bus	6.5	3	4	9.5	10.5
Truck	6.0	5	6.5	11.0	12.5
Tractor-trailer	7.5	6.5	8.0	14.0	15.5

units. The minimum holding times for left-hand and right-hand toll booths can be computed as follows:

Left-Hand Booths

$$\begin{aligned} \text{H.T.} &= 0.64\times 8+0.15\times 9.5+0.14\times 11+0.07\times 14. \\ &= 5.1+1.4+1.5+1.0=9.0 \text{ sec.} \end{aligned}$$

Maximum booth capacity = 3600/9 = 400 vehicles/hour.

Right-Hand Booths

$$\begin{aligned} \text{H.T.} &= 0.64\times 9+0.15\times 10.5+0.14\times 12.5+0.07\times 15.5. \\ &= 5.8+1.6+1.8+1.1=10.3 \text{ sec.} \end{aligned}$$

Maximum booth capacity = 3600/10.3 = 350 vehicles/hour.

As a check against this method, another method was utilized. At the George Washington Bridge during 18 peak periods in which there was heavy congestion due solely to overloaded toll booths, the average traffic per lane was 403 vehicles. Assuming a 95 per cent occupancy at these times, a minimum holding time of $0.95\times(3600/403)=8.5$ was indicated. This compares exactly with the results of the toll-time and vehicle-time analysis with equal numbers of left- and right-hand booths handling a composition of traffic consisting entirely of passenger cars, which was virtually the composition at the George Washington Bridge on the occasions mentioned. This method is applicable only at a bridge, because at a tunnel entrance the congestion caused by the tunnel itself during peak traffic periods prevents traffic from moving out of toll booths when the transaction is over, thus artificially lengthening the holding time, and at a tunnel exit the tunnel holds back traffic, thus preventing saturation of the toll booths.

DEVELOPMENT OF AVERAGE DELAY CURVES

Having established the relation of traffic per lane vs holding time, in addition to the relation of percentage of occupancy vs delay ratio, it is now practical to develop the relation of traffic vs average delay in seconds that was originally sought. Table V shows sample computations of points for a curve for four left-hand toll booths using values taken from the previous two types of curve. Table V applies to the George Washington Bridge only, since the holding-time values given in the third column apply only to this facility. These points were computed by first assuming a traffic volume per lane, and working from there. Take the point given by 300 vehicles per lane per hour. The next column shows the total traffic volume of 1200 for the four lanes. The third column gives the booth holding time, 9.8 sec, which was read from the holding-time curves. The booth holding time multiplied by the vehicles per lane gives 2940 booth-seconds of traffic per lane shown in the fourth column. Dividing the latter value by the 3600 booth-seconds available in 1 hr gives the 81.6 per cent occupancy shown in the next column. Entering the occupancy-delay-ratio curve for four left-hand toll booths at this occupancy gives a delay ratio of 1.31, and multiplying the delay ratio by the holding time of 9.8 sec yields an average delay value of 12.8 sec.

TABLE V

TRAFFIC VS DELAY POINTS FOR FOUR LEFT-HAND BOOTHS

Vehicles per lane per hour	Total vehicles	Holding time, sec	Booth-seconds	Occupancy, %	Delay ratio	Delay, sec
100	400	12.9	1290	36.0	0.20	2.6
150	600	12.7	1910	53.0	0.45	5.7
200	800	11.8	2360	65.5	0.73	8.6
250	1000	10.8	2700	75.0	1.02	11.0
300	1200	9.8	2940	81.6	1.31	12.8
350	1400	8.9	3120	86.7	1.66	14.8
375	1500	8.7	3240	90.0	2.00	17.3
385	1540	8.6	3300	91.7	2.36	20.2
400	1600	8.5	3400	94.4	3.40	29.9

When points had been computed and plotted for all the various booth combinations generally used at the George Washington Bridge, the result was a family of curves, as shown in Fig. 8. From these curves, it is apparent that the traffic-carrying capacity of different toll booths for a given delay is not constant but instead varies appreciably between different combinations of booths for a given amount of delay. Before this analysis was made, it was generally assumed by the management in scheduling manpower that one toll booth was just about like another in all circumstances. Again the overflow nature of the right-hand toll booths shows up here.

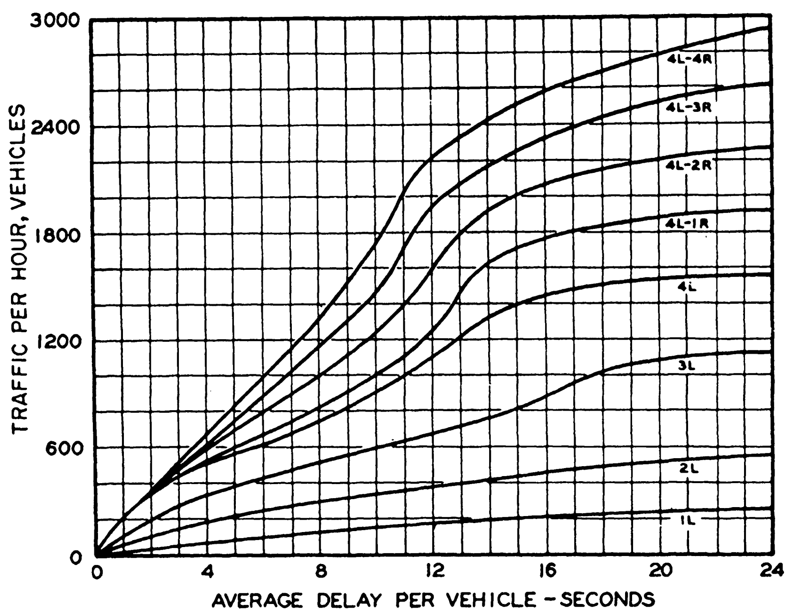

FIG. 8. Average delay for various volumes of traffic at George Washington Bridge.

The curves for combinations of four lefts with one to four rights all merge into the curve for four left-hand booths alone at a volume of about 400 vehicles per hour. Below this volume the right-hand booths carry virtually no traffic.

One solution to the delay problem has now been achieved, but, before it is used, some indication of its accuracy would be desirable. To see whether the curves actually portrayed what was given by the original observations, values read from the curves were compared with the direct computations of average delays from the data. In so doing it was found that for observation periods of approximately 20 min the average error was 2.64 sec. Considering that the values computed from the data represent the mean of a sample of the population, it can be estimated that for a sample of three times this size the probable error would be less by a factor of one to the square root of 3, making it 1.53 sec. The average delay of all observations was 11 sec, thus indicating that for hourly periods the curves would predict average delay with a probable error of about 15 per cent. This, fortunately, was close enough for our purposes. If it had not have been, we would have had to turn to some other criteria that could be predicted for purposes of setting service standards and determining how many toll booths are required for various volumes of traffic.

ANALYSIS OF TRAFFIC BACKUP

Very often in waiting problems, knowledge of the average delay involved is insufficient. An analyst is interested in this, but he is also interested in the boundary conditions of what the worst delays might be under given circumstances. If, on the occasion of an important appointment, a motorist is delayed several minutes waiting in a line of many vehicles to get through the toll booths, he would likely find little consolation in being told that by using Port Authority facilities regularly he could expect his average delay to be very nominal. This realization leads to an analysis of traffic-backup behavior.

One way of analyzing backup behavior is simply to plot values of the greatest backups observed against the traffic volume handled for each combination of toll lanes. When this is done, the problem of wide scattering again arises. For this particular relation the scattering is worse than for any other investigated. From the limited amount of data taken, only the roughest idea can be obtained of what maximum backup to expect and how often to expect it for a given combination of toll booths handling a given volume of traffic. It is therefore necessary to employ the methods of mathematical statistics to determine the relation.

In organizing and summarizing the data shown in Table I for purposes of statistical analysis of backup behavior, it was decided to consider the number of vehicles in the longest waiting line, rather than the total amount of backup behind all open toll booths. The reason, of course, is that we are concerned with the one motorist who incurs the worst delay, and total backup is not a measure of this because of the nonuniform distribution of backup between the open lanes.

The first steps in the analysis of backup are similar to those taken in the analysis of traffic arrivals. One difference, however, is the use of much smaller samples. In the traffic-arrival analysis, the data were grouped into 200-vehicle volumes. By so grouping, samples consisting of a few hundred intervals could be obtained, and the frequency polygons resulting from the samples were fairly smooth. In studying traffic arrivals, consideration did not have to be given to the number and types of toll booths employed, but, for the backup analysis, observations have to be so segregated. The toll-booth arrangements are, in practice, changed two or more times an hour because of changing traffic volume and because the reliefs given to toll collectors sometimes result in a booth of one type being substituted for one of the other type. It is therefore expedient in analyzing backup behavior, as was also the case in the average delay analysis, to use periods of about 20 min. This provides samples of only about forty intervals—two a minute for 20 min. To smooth out the irregularities in the frequency distribution resulting from the small samples, a three-point weighted moving average can be employed.

Figure 9 shows the results obtained in plotting frequency distributions of the backup in the longest line for a combination of three left-hand toll booths, after the observed distributions had been smoothed by averaging and had been converted to a base of 100 to give frequency as a percentage of total occurrences. These curves include cases from both the Lincoln Tunnel and the George Washington Bridge. The first two curves, for volumes of 575 and 670 vehicles per hour, are from the tunnel, and the other two, for volumes of 705 and 890 vehicles per hour, are from the bridge. It will be noted that the distributions resemble the traffic-arrival distributions, as one might expect, since holding times are essentially constant in distribution and therefore the cause of variations in backup is largely the variation in traffic arrivals.

Figure 10 shows Poisson distributions corresponding to the actual distributions shown in Fig. 9. Except for the irregularities of the actual distributions, they resemble the Poisson distributions. In computing values for the Poisson distributions, the same mean value of backup found in the actual distributions was employed. This feature is different from the traffic-arrival analysis since in the latter it was unnecessary to determine the mean value empirically; it is given directly from the traffic volume and observation interval. There is no doubt a definable relation between traffic volume and the mean value of backup for a given booth combination, but we could develop no formula, either theoretical or empirical, that would predict the mean value of backup for a given volume of traffic.

How closely the Poisson distribution fits the actual distribution of backup is illustrated more clearly in Fig. 11, which shows both distributions plotted together. This case covers a condition of three left-hand toll booths handling traffic at the rate of 615 vehicles per hour at the Lincoln Tunnel. This, incidentally, portrays the values given in the sample data presented in Table I. For this case, the mean value of backup is 2.16 vehicles, and the standard deviation is 1.52 vehicles. The standard

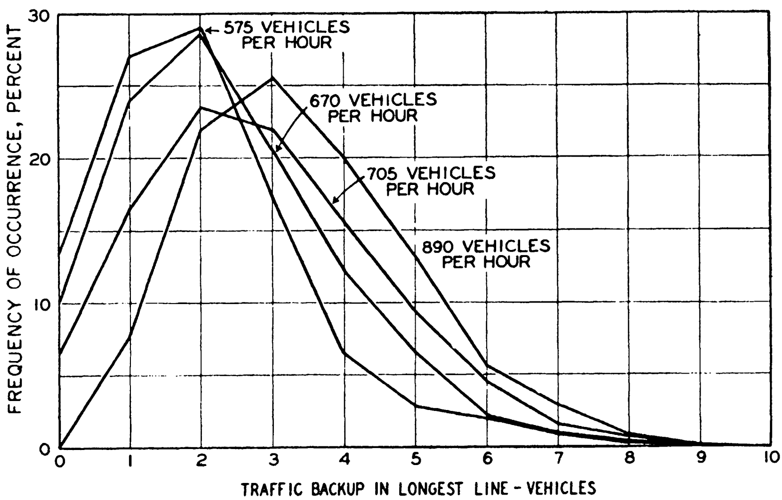

Fig. 9. Actual frequency distribution of traffic backup for three left lanes.

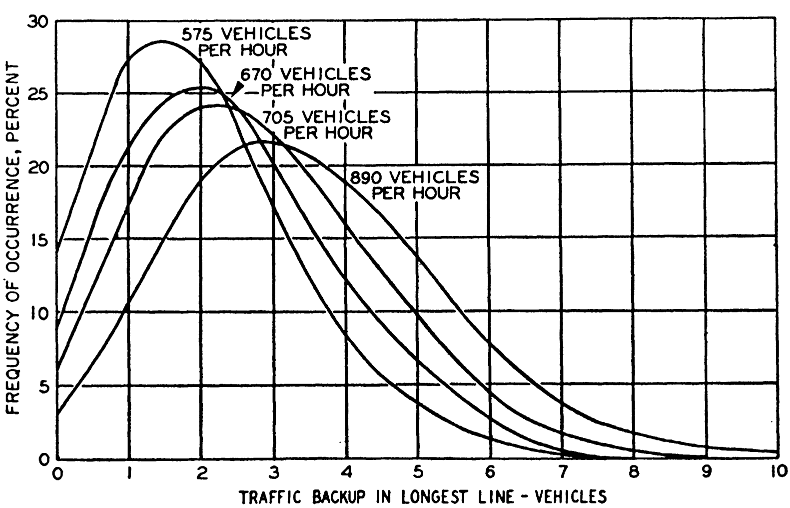

Fig. 10. Theoretical frequency distribution of traffic backup for three left lanes.

deviation of the mean, which will be used later, is 0.15. The chi-square probability level for the Poisson is 0.64. For a normal distribution the chi-square probability is only 0.01.

The results at the George Washington Bridge are comparable to those at the Lincoln Tunnel, as shown in Fig. 12. This is at a slightly higher volume of 705 vehicles per hour, and the mean backup value is 2.79 vehicles, the standard deviation is 1.67, and the standard deviation of the mean is 0.31. The chi-square probability level (used here as a rough indicator

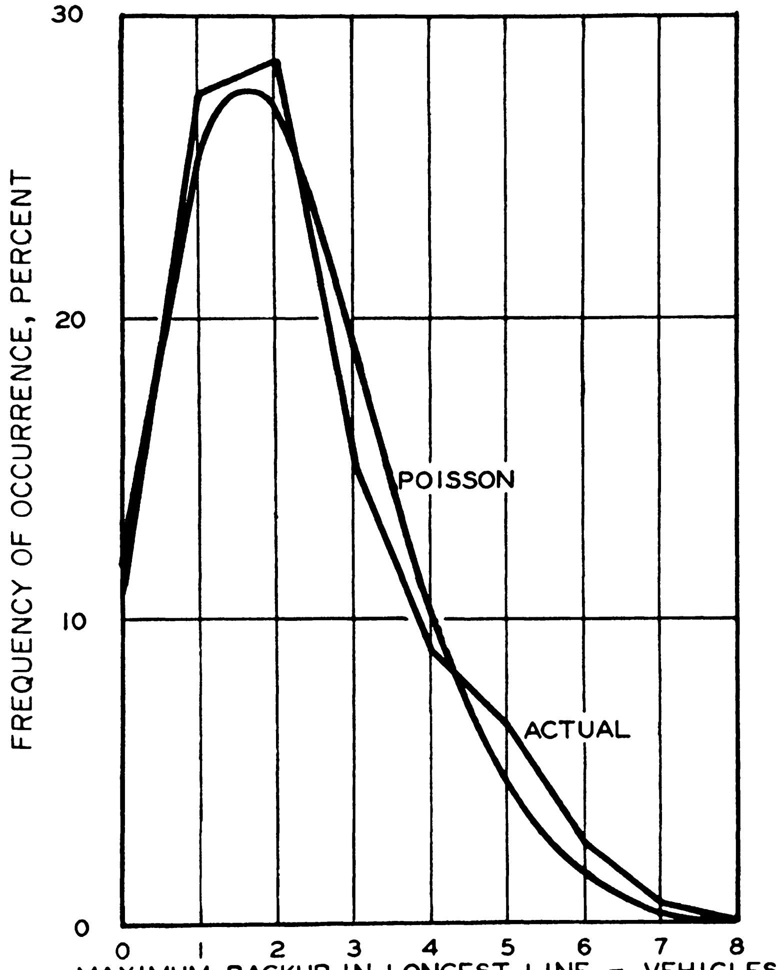

FIG. 11. Actual and theoretical backup for 615 vehicles per hour in three left lanes at Lincoln Tunnel.

of goodness of fit) for the Poisson is 0.55, and, for a normal distribution, less than 0.01.

In the same way that the traffic-arrival patterns at the tunnel and the bridge are nearly identical, the backup behavior is also nearly identical. The identity is so close that we were quite unable to differentiate between the two facilities. This was rather surprising since there were quite discernible differences in average delay values between facilities because of differences in traffic composition and holding time. We spent a considerable amount of effort trying to find differences in backup values, but without success. It was decided that, except for conditions approaching satu-

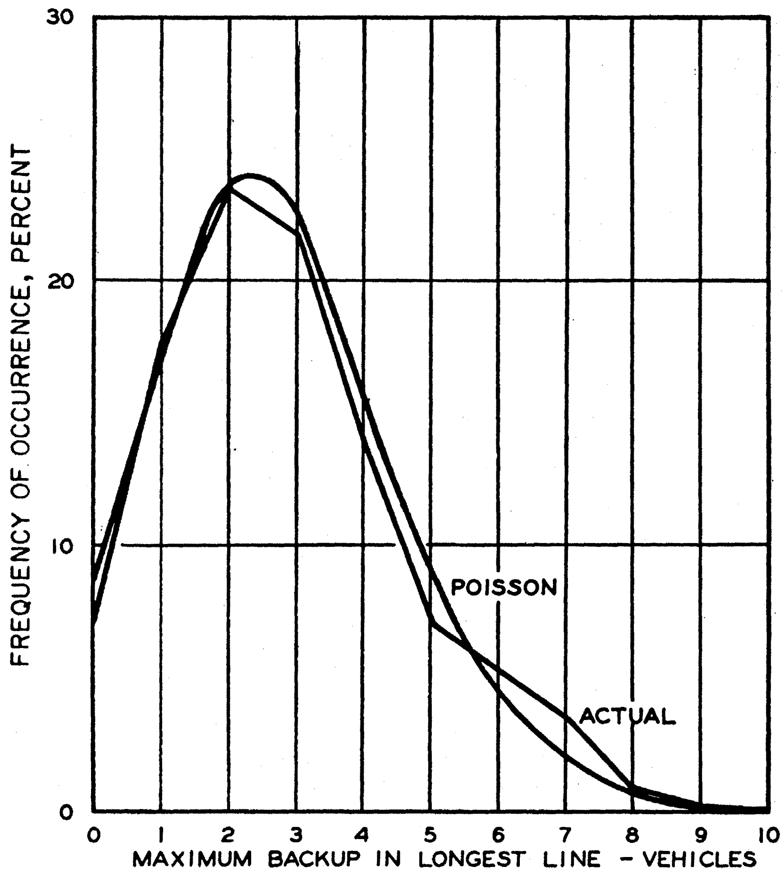

FIG. 12. Actual and theoretical maximum backup for 705 vehicles per hour in three left lanes at George Washington Bridge.

ration, the greater amount of backup caused by a longer holding time for a given traffic volume is reflected more in time units than in vehicle units. As a specific illustration, the mean value of backup for 615 vehicles per hour and three left-hand toll booths was found to be 2.16 vehicles, and the booth holding time was 11.1 sec. This represents a backup in time units of $2.16 \times 11.1 = 24.0$ sec. If the holding time increased say 20 per cent to

13.3 sec and the time backup also increased 20 per cent to 28.8 sec, the vehicle backup would remain the same at 2.16. Something close to this seems to happen for small differences in holding time.

In all cases of backup distribution, the normal showed a poorer fit than the Poisson so that the latter can be considered the true nature of the backup distribution—up to a point. Table VI indicates that the Poisson distribution does not hold indefinitely as traffic volumes are increased. Starting with a rather remarkable fit of 0.93, at a volume of 575 vehicles per hour, the goodness of fit drops off gradually reaching an unsatisfactory value at a traffic volume of about 800 vehicles per hour. This particular volume applies only to three left-hand toll booths, but the same deterioration of fit was observed at other volumes for all toll-booth combinations as the traffic volumes approached values of approximately 60 to 75 per

TABLE VI

Relation of Goodness of Fit of Backup to Poisson Distribution for Three Left-Hand Toll Booths

Traffic volume	Goodness of fit
575	0.93
615	0.64
625	0.55
670	0.85
705	0.55
750	0.05
867	0.01
890	0.32

cent of saturation. The reason for this deterioration appears to be the increasing carry-over of vehicles from one interval to the next as saturation is approached. The traffic volume at which the Poisson distribution broke down was termed the 'Poisson point.'

TRAFFIC VOLUME VS MEAN VALUE OF BACKUP

Having established the range of usefulness of the Poisson distribution, the next step is that of establishing the relation between traffic volumes and the mean value of backup, the mean value being the sole parameter necessary to specify the entire distribution. The only satisfactory method found to determine mean values was to draw an empirical curve, as shown in Fig. 13. To assist in locating the curve, the points were plotted to show plus and minus one standard deviation of the mean. In many cases, as in this one, most of the points were more or less clustered within the range of traffic volumes customarily handled by the booth combination concerned. To obtain empirical values at higher traffic volumes would have required the deliberate creation of excessive congestion, which would make some patrons very unhappy. Fortunately, this is unnecessary because it is obvious the curves approach the full occupancy capacity of the booth combination asymptotically. Full occupancy capacity is known to be approximately 400 vehicles per hour for left-hand booths at the Lincoln Tunnel and 450 at the George Washington Bridge. To be on the safe side, the lower value was used, and the curve for three left-hand booths was drawn to approach a volume of 1200 vehicles per hour.

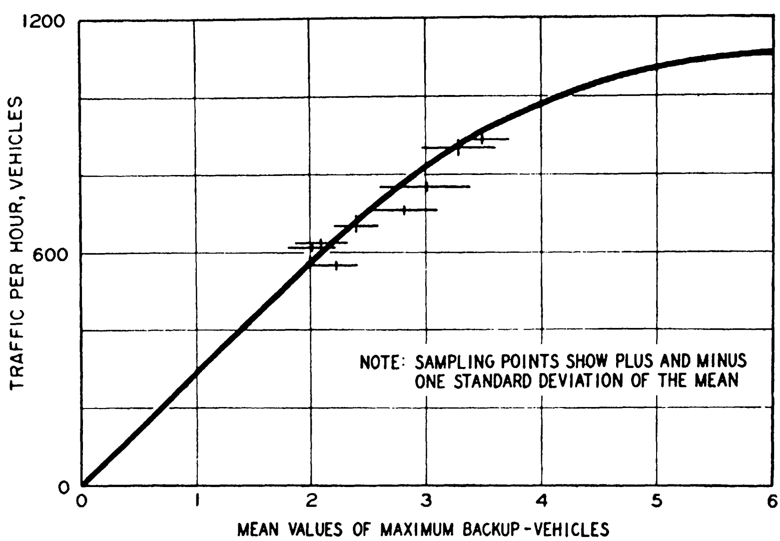

FIG. 13. Mean values of maximum backup for three left-hand toll booths.

Combining similar curves for various toll-booth combinations results in the family of curves shown in Fig. 14. When the Poisson points were plotted on this chart they were found to be very nearly in the straight line shown dotted and labeled the 'Poisson line.'

PROBABLE MAXIMUM BACKUP

Knowledge of the mean values of backup in the longest line, plus knowledge that the distribution of this variable is Poisson, permit us to investigate the boundary values, which can be determined by Poisson summations. The question is what boundary values are we interested in? Or in telephone terminology, what loss probability should be used? The answer to this question depends somewhat on judgment. If the loss probability is made too large, say 0.1, the boundary value would be exceeded so frequently that it would be a poor measure of the maximum delay that a patron would incur. On the other hand, if the loss probability is made too small, say 0.001, the boundary condition would occur so infrequently as to be misleading in the other direction. Looking at this question in a slightly different way, if we consider the period of interest to be 1 hr composed of 120 30-sec intervals, and the boundary value of backup is taken as that having a probability of 0.1 of being equaled or exceeded, the expected number of intervals of occurrence would be $0.1 \times 120 = 12$. If the loss probability is taken as 0.001, then the expected number of intervals that it would occur in 1 hr would be 0.12, and the maximum would be ex-

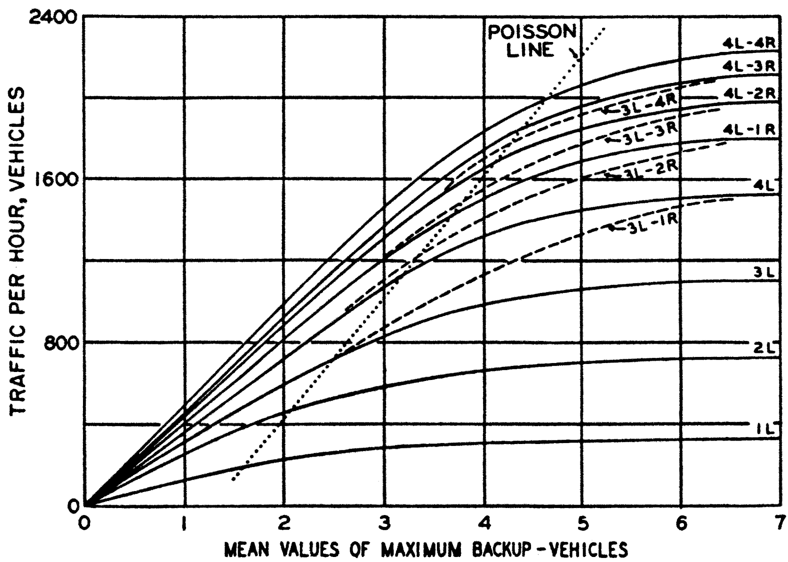

FIG. 14. Mean values of maximum backup.

pected to occur only once in over 8 hr. It therefore appears logical to choose a loss probability of 0.01 to define what was called the 'probable maximum backup,' since this maximum could be expected to occur or be exceeded for about one 30-sec interval out of an hour. The family of curves shown in Fig. 15 gives backup values at that probability level. These curves have again been extrapolated considerably beyond the data, and the extrapolation has been made by approaching asymptotically traffic volumes of 400 times the number of toll booths.

As in the case of the average delay curves it was considered advisable to establish the reliability of the probable maximum backup curves by a comparison with observed values. For some 53 periods of observation at the Lincoln Tunnel and the George Washington Bridge covering from one to eight booths, over a period of about 20 hr, there were 26 individual observations out of 2379 in which the actual maximum backup equaled or exceeded the probable maximum backup given by Fig. 15. The probability of equaling or exceeding values read from the curves indicated by this is 26/2379 = 0.0109, an error of 9 per cent from the objective of establishing the p (0.01) values. Out of the 53 periods, which averaged about 20 min each, there were 10 periods in which the backup exceeded the prediction for one or more intervals of 30 sec. The average excess amounted to 1.4 vehicles and the maximum was 4. For 41 intervals the actual maximum was less by an average amount of 1.5 vehicles and a maximum of 4.

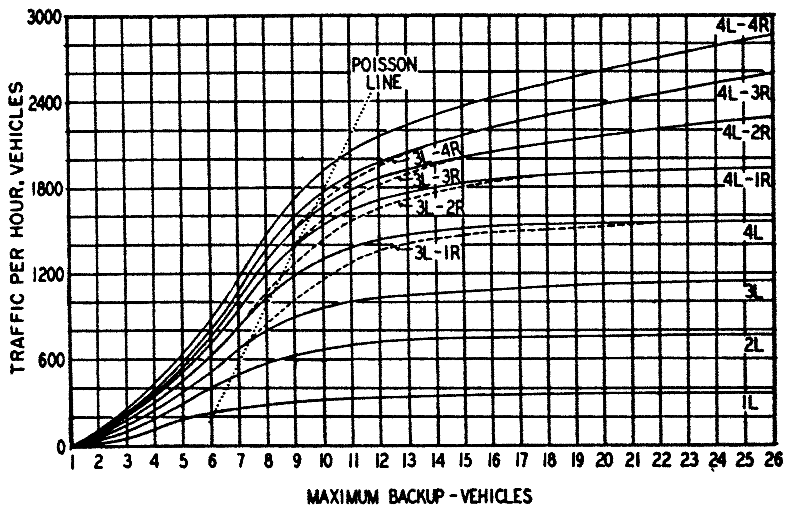

FIG. 15. Probable maximum backup in longest line.

THE OPTIMIZING PROBLEM

Having solved the waiting-line problem in terms of average delay per vehicle and probable maximum backup, the next problem is that of establishing an optimum level of service, i.e., of setting service standards. One way is to select an upper limit of average delay, such as 20 sec, and to open another toll booth when this limit is reached. Such an arbitrary decision is difficult to support, and is hardly to be recommended in operations research. Furthermore, the objective is not so much to place an upper limit on delay as it is to control delay more closely than had been done in the past. The principal dissatisfaction with former methods of manning toll booths was that they varied so widely—from less than 2 to nearly 50 sec under substantially normal off-peak conditions.

In order to reduce the extreme swings of average delay and at the same time to optimize the service it is suggested that the standard should be a middle value rather than a maximum. As traffic increases, the average delay should swing above the standard by an amount equal to the drop below the standard when an additional booth is manned. The question is how to select this middle value of delay in a logical manner with a minimum of arbitrariness.

One way would be to assign relative values to traffic volumes handled and serviced; for example, let 10 vehicles per booth-hour be considered equivalent in value to an increase in delay of 1 sec. This method of equating would be hard to support logically. A better one along the same line would be to consider patron time and toll-collector time of equal value. Thus another booth would be opened when the traffic volume times the reduction in delay to be achieved by another booth would equal 3600 sec. Although this principle makes a certain amount of sense, it was not used.

Another way is to consider the point of diminishing returns. This method has the advantage of being less controversial and comprises a concept widely accepted and understood by management people. In this case the cost is characterized by the delay and the return by the traffic volume. The point where return starts diminishing in relation to the cost is that of minimum curvature of the curves. Above this point the increases in traffic volume attained for each increment of increase of delay becomes smaller and smaller, approaching zero as the delay approaches infinity.

The points of diminishing return defined in this way can be determined by inspection. For the George Washington Bridge they vary from 10½ to 16 sec with a weighted average of about 12 sec. For the Lincoln and Holland tunnels, they average about 10 and 11 sec respectively. Since it was desired to provide uniform service at the three crossings, the middle value of 11 sec was adopted as the standard for all three.

Now capacity standards can be established for the various groups of toll booths by equalizing the swing on each side of the standard delay as additional booths are provided. Doing so at the George Washington Bridge resulted in Table VII. It will be noted that the backup values at the booth capacities for the standard average delay increase as the number of booths are increased, ranging from 6 with one booth to 12 with eight booths. Fortunately, this is a desirable result since experience has shown that patrons are more willing to accept longer lines as traffic volumes increase. Apparently they intuitively feel that a backup of 12 cars when eight toll booths are open is qualitatively different from a backup of 12 cars with only one toll booth open.

HOURLY TRAFFIC PATTERN

At this point, two service criteria—average delay and maximum backup—can be satisfactorily predicted when the traffic volume is known, and a standard for one has been established. The question that next arises is how well traffic volumes can be predicted. This question requires a study of the hourly pattern of traffic throughout the day and the dispersion from day to day. This analysis was made by plotting hourly volumes on charts

TABLE VII
GEORGE WASHINGTON BRIDGE TOLL-BOOTH CAPACITY

Booths		Capacity, vehicles/hr	Delay, sec	Backup, number of cars
Left-hand	Right-hand			
1		225	0–16.9	6
2		450	5.1–15.5	7
3		750	6.5–14.0	8
4		1050	8.0–11.5	8
4	1	1250	10.5–12.0	9
4	2	1525	10.0–11.8	10
4	3	1850	10.2–11.5	11
4	4	2150	10.5–11.5	12

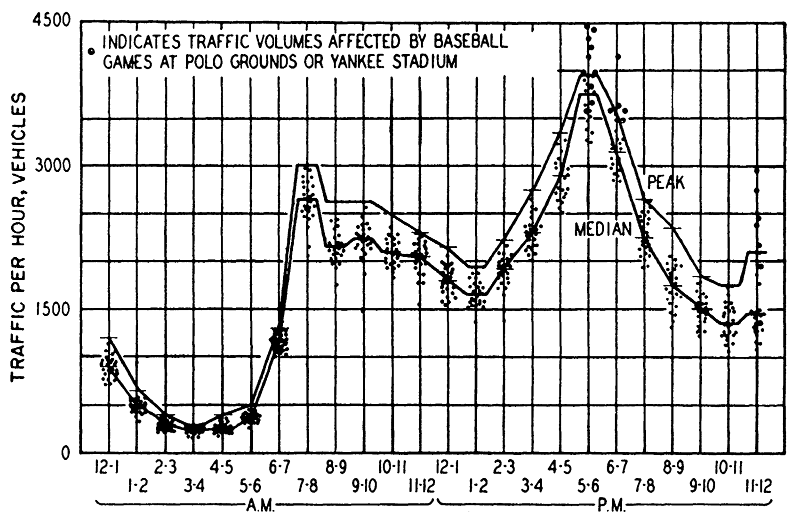

FIG. 16. Hourly volume of westbound traffic on George Washington Bridge.

having time of day as abscissas and traffic volumes as ordinates. In making this analysis it was found that the days in the middle of the week had almost identical patterns and could be combined. Figure 16 shows a pattern found for Tuesdays, Wednesdays, and Thursdays combined at the George Washington Bridge for the summer of 1952. On the other days of the week each day was so different that it required separate treatment.

As can be seen in Fig. 16 two curves were drawn through certain of the plotted points. One curve was drawn through median points, which was the simplest way of obtaining an estimate of expected values of traffic without many computations. Another curve was drawn through the peak values as the simplest means of estimating the highest values of traffic to be expected.

Inspection of the curves indicates a spread between median and peak figures of from 10 to 60 per cent at the George Washington Bridge. At the tunnels the spread was less, ranging from 10 to 30 per cent. These variations limit how closely toll booths can be scheduled in advance to provide optimum service, which brings us to the last part of the problem, the scheduling of toll booths and collectors.

THE SCHEDULING PROBLEM

In the scheduling of toll booths throughout a day, the number of booths required was first determined from the capacities of various booth combinations derived on the basis of optimum average delay for the median traffic volumes. Because of the rapid rise and fall of traffic at daily peaks, it was necessary to do this by half hours. When done, the peak values of traffic for each half-hourly period were studied for the maximum backup that might occur. Concern was then given to those cases where maximum backups several vehicles above the Poisson points were indicated. Our ability to predict backups satisfactorily no doubt fell off rather rapidly in this region. Since saturation of booth capacity was being approached, traffic volumes slightly higher could cause a significant jump in backups. Judgment was used here to determine how much of a gamble to take. Although more precise methods could be used, they were unnecessary. Judgment suggested a gamble on backups up to three vehicles above the Poisson points. Therefore, when the spread between median and peak traffic was great enough to result in backups exceeding this standard, an additional booth was provided.

This process resulted in a schedule of booths throughout the day, from which could be determined the total number of booth-hours required for the day. One more step remained in the problem, that of determining how many toll collectors were required to keep the scheduled number of booths open, and still permit toll collectors' personal and meal reliefs to be given within certain restrictions. These restrictions were (a) working periods of not less than 1 nor more than 3 hr between reliefs or ends of the collector's tour, (b) meal reliefs in the middle 4 hr of an individual's tour, and (c) starting times not earlier than 6 A.M. and quitting times not later than 12:30 A.M.

The scheduling of manpower in such a manner requires the preparation of a Gantt-type chart for each day, showing the working and idle periods for every toll collector. Toll-collector starting times and relief periods must be juggled in an effort to provide exactly the number of collectors needed to give the optimum service each half hour of the day. This is largely a trial-and-error problem, and preparation of such schedules may be very time-consuming when the objective is to make the schedule as efficient as possible.

The efficiency of such a schedule is given by the ratio of the number of collectors required by the booth-hours to the number supplied by the schedule. As an example, the mid-week days at the George Washington Bridge during the summer of 1953 required 344 booth-hours per day. The net working time per toll collector per day is 6¼ hr; thus the minimum number of collectors that would meet booth-hour requirements is $344/6.25=55.04$. If a schedule uses 57 men, its efficiency is $55.04/57=97$ per cent. The first schedules made were not very efficient, and there was always a question whether a given schedule was the most efficient that could be made as long as the number of collectors used exceeded the first integral number above requirements. A great deal of time can be wasted in trying to reduce the number of collectors employed, when it actually is not possible to do so within the restrictions imposed.

Analysis and experience show, however, that the efficiency of such a schedule depends largely on the magnitude and duration of peak periods. By considering the relief requirements during the morning and evening peaks, and the period just after midnight, an estimate can be made of the number of collectors required on each tour. This analysis is made by totaling the number of booth-hours required for the peak 3½ hr and dividing by 3. Doing so allows a one-half-hr relief period for each toll collector. Continuing with the example of the George Washington Bridge mid-week days, there are 70 booth-hours in the morning peak, requiring $70/3=23.3$, or 24, men; 71.5 booth-hours in the evening peak, requiring $71.5/3=23.8$, or 24, men; and 21 booth-hours after midnight, requiring $21/3=7$ men. The total for the three tours comes out to 55 men, thus indicating a schedule close to actual requirements is possible. The actual schedule used 56 men for a scheduling efficiency of $55.04/56=98.3$ per cent. In most cases traffic patterns enable scheduling efficiencies of 95 per cent or better.

RESULTS

With the development of an efficient method of scheduling, the last problem of the study was solved. A big question, however, remained before the results could be recommended to management. This question was: would a method of manning toll booths based on these techniques really work any better than the former method of just giving a toll sergeant approximately the right number of collectors and letting him use his own judgment about how many booths should be kept open as traffic varied and when collectors should be given reliefs. The only way to find out was to try it. If it worked continuously for a week, it should be able to work indefinitely.

A trial was conducted at the Lincoln Tunnel. The numbers of toll booths required every half hour for the entire week were predicted in both directions of traffic. This entailed 512 predictions of booth requirements. Each toll collector was given a slip showing his booth assignments and relief periods and was instructed to follow the schedule strictly. During the entire week, the prearranged schedules were followed without a hitch. At no time did excessive backups occur, and at no time did reliefs have to be deferred. The movement of collectors and the opening and closing of booths took place without the attention of the toll sergeant. At times the number of booths were slightly excessive, but not to the extent previously occurring under the former method. Needless to say, there is a good deal of satisfaction in seeing the validity of so much work actually established.

LITERATURE CITED

1. E. Brockmeyer, H. L. Holstrom, and Arne Jensen, *The Life and Work of A. K. Erlang*, Copenhagen Telephone Co., Copenhagen, Denmark, 1948.
2. G. S. Berkeley, *Traffic and Trunking Principles in Automatic Telephony*, Ernest Benn Ltd., London, 1949.
3. C. D. Crommelin, *P.O. Elec. Engrs. J.*, **26**, Pt. 4, Jan. 1934.
4. B. D. Greenshields, D. Shapiro, and E. L. Erickson, "Traffic Performance at Urban Street Intersections," Yale University, 1947.