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Author(s): George B. Dantzig

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A COMMENT ON EDIE'S "TRAFFIC DELAYS AT TOLL BOOTHS"

GEORGE B. DANTZIG

The Rand Corporation, Santa Monica, California

(Received June 21, 1954)

E. W. PAXSON of Rand has suggested that linear programming methods could be used as an alternative procedure in the scheduling section of Leslie C. Edie's interesting paper on "Traffic Delays at Toll Booths."* The purpose of this note is to elaborate on this suggestion.

Edie, in earlier sections of his paper, develops a method for determining booth requirements; he then states the scheduling problem as follows:

"This process resulted in a schedule of booths throughout the day, from which could be determined the total number of booth-hours required for the day. One more step remained in the problem, that of determining how many toll collectors were required to keep the scheduled number of booths open, and still permit toll collectors' personal and meal reliefs to be given within certain restrictions. These restrictions were (a) working periods of not less than 1 nor more than 3 hr between reliefs or ends of the collector's tour, (b) meal reliefs in the middle 4 hr of an individual's tour, and (c) starting times not earlier than 6 A.M. and quitting times not later than 12:30 A.M."†

We propose first to discuss the scheduling under the simplified assumption that the assignments are made ignoring the need for reliefs. (Later on we will say a few words about reliefs and an important variation of the procedure.) The effective day for each toll collector is given as $6\frac{1}{4}$ hr. However, including his relief, it will be supposed that it becomes a 7-hr effective day which has a standard pattern of $3\frac{1}{2}$ hr before the meal, $\frac{1}{2}$ hr for the meal, and $3\frac{1}{2}$ hr after the meal. If now we let x_i be the number of men who start their tour at time j , where $j=0, 1, 2, \dots, n$ corresponds to the half-hour intervals throughout the day and if we let $a_{tj}=1$ if $t=j, \dots, j+6, j+8, \dots, j+14$ (where $j+7$ corresponding to the meal period is omitted) and $a_{tj}=0$ otherwise, then the assignments must be chosen such that

$$\sum_{j=0}^n a_{tj} x_j \geq b_t, \quad (t=1, 2, \dots, n+14) \quad (1)$$

where b_t are the required number of toll booths for period t . The optimum assignment problem may then be stated as finding a set of non-negative x_i satisfying (1) such that the total number of men assigned is a minimum, i.e.,

$$\sum_{j=0}^n x_j = \text{Min.} \quad (x_i \geq 0) \quad (2)$$

Computationally, the writer has discovered (in another connection) that the solution to system (1) and (2) can often be done by hand in a short time (1 to 2 hours) because the $a_{tj}=0$ or 1. (A special adaptation of the dual form of the

* LESLIE C. EDIE, "Traffic Delays at Toll Booths," *J. Opns. Res. Soc. Am.* **2**, 107 (1954).

† *Ibid.*, p. 136.

simplex algorithm* is employed that does not require knowledge of the inverse of the basis. Since the special procedure is unpublished, I will be happy to supply readers with further details if they write to me.)

Reliefs, other than meals, may be treated by introducing more alternative patterns of work with time-period gaps introduced into the pre-meal and post-meal reliefs. (This may necessitate going to 15-minute instead of half-hour intervals.) The effect would be to multiply the number of unknowns by k^2 where k is the number of different times when a relief can take place in the pre- or post-meal part of the tour. Mathematically, reliefs cause an increase in the number of unknowns in system (1) and (2) while preserving its essential structure. This need not add greatly to the work. For example, one could optimize using only a subset of the patterns, e.g., using only a relief in the middle of the early and late part of the tours and then (dropping the dual approach) introduce the other variables and their associated work patterns as a refinement of the restricted solution.

Fractional values for the optimum x_i are possible, although in the few cases tested this did not occur. Roundings (some up, some down) will produce a solution with the value of (2) larger than the minimum for the number of men required. Their difference is likely to be small. Only if the difference is large should there be any concern about accepting the rounded solution as the true optimum.

By slightly reformulating the concept of work patterns, it is possible to reduce this problem to a variant of the standard transportation-type† linear programming problem. Here, special procedures exist that permit solving very large systems rapidly. It has the following advantages: (a) The solutions will be in integers. (b) The work pattern idea is more general, inasmuch as the worker is not forced to fit the work pattern exactly if he is not needed. To illustrate, consider the following array:

Pattern	Period t						Total
	(1)	(2)	(3)	(4)	(5)	(6)	
1.1	x_{11}	—	x_{13}	x_{14}	—	x_{16}	w_1
1.2	x_{21}	—	x_{23}	x_{24}	—	x_{26}	w_2
2.1	x_{31}	x_{32}	—	x_{34}	x_{35}	—	w_3
2.2	x_{41}	x_{42}	—	x_{44}	x_{45}	—	w_4
2.3	x_{51}	x_{52}	—	x_{54}	x_{55}	—	w_5
3.1	—	x_{62}	—	x_{64}	x_{65}	—	w_6
—	—	—	—	—	—	—	w_0
Total	b_1	b_2	b_3	b_4	b_5	b_6	N

The work pattern 1.1 states that a worker has reliefs in periods (2) and (5). In the other periods he must work only if needed at a toll booth, i.e., if $x_{it}=1$ means

* GEORGE B. DANTZIG, ALEX ORDEN, and PHILIP WOLFE, "The Generalized Simplex Method for Minimizing a Linear Form under Linear Inequality Restraints," Rand Corporation, RM-1264, 5 April 1954; GEORGE B. DANTZIG, "The Dual Simplex Algorithm," Rand Corporation, RM-1270, 3 May 1954.

† GEORGE B. DANTZIG, *Activity Analysis of Production and Allocation*, "Application of the Simplex Method to a Transportation Problem," Koopmans (Ed.), 1951.

he works in the period t , $x_{it}=0$ means he does not. Each work pattern, e.g., (1.1 and 1.2) is repeated a number of times since several workers may be assigned the same pattern. The number of repetitions must be guessed in advance; the only requirement being that it should be in excess of any likely number of similar assignments found in an optimum solution. The equations requiring solution are

$$\sum_i x_{it} = w_i, \quad \sum_i x_{it} = b_t, \quad \sum_i w_i = N,$$

where x_{it} is omitted if (i, t) corresponds to a relief period, where w_i is the number of time periods in a work pattern actually needed, b_t is the total number of toll gates required in period t , N is the number of time periods available in a given day from the entire work force, and w_0 is the amount not used. The variables x_{ij} , w_i must be chosen such that

$$1 \geq x_{ij} \geq 0, \quad w_i \geq 0, \quad w_0 = \text{Max},$$

i.e., the variables are so chosen that the force will be as free as possible for other work. It will be noted that the condition of $x_{ij} \geq 1$ is an upper bound on a variable. Here again a variant of the simplex algorithm exists which takes advantage of this type of condition.*

* GEORGE B. DANTZIG, "Variables with Upper Bounds in Linear Programming," Rand Corporation, RM-1271, 22 March 1954.

WAITING LINE SUBJECT TO PRIORITIES

JULIAN L. HOLLEY

Melpar, Inc., subsidiary of Westinghouse Air Brake Company, Alexandria, Virginia
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IN HIS ARTICLE, "Priority Assignment In Waiting Line Problems,"* Alan Cobham obtained very interesting and valuable waiting-time formulas both for single- and multiple-channel systems. His reasoning is similar in the two cases, but it is given in detail for the single-channel case, and for definiteness the ensuing remarks will be directed exclusively to that case.

In view of the importance of Cobham's result, it seems desirable to amplify the discussion in two respects. In the first place, we shall sketch an alternative approach that avoids his infinite iterative procedure and that applies the relevant probability principles in what appears to be the simplest possible way. In the second place, we shall examine the implications of Cobham's formula for the case of a single priority level, i.e., for the case with which the literature is almost exclusively concerned.

To avoid needless duplication we shall use at will the definitions and notation of the Cobham article (*q.v.*), and in addition we shall introduce the quantities

$$\psi_k = \lambda_k / \mu_k, \quad (1 \leq k \leq p)$$

$$\sigma_k = \sum_{i=1}^k \psi_i, \quad (0 \leq k \leq p) \quad (2)$$

* COBHAM, *J. Opns. Res. Soc. Am.* **2**, 70 (1954).