Service Engineering

Homework 8 -
Service Processes and Analysis of Customers' Patience.

Submit questions:

- Part 1: 2,3 and 4.
- Part 2: 1,2 and 3.

Part 1. Theoretical and Empirical Analysis of Service Times.

Question 1. The following graphs are based on the Call Center data from a small Israeli bank. The service times during November and December were analyzed, and the four major service types (PS, IN, NE, NW) were considered.

Figure 1 presents the survival functions of service times for the four types. The points in Figure 2 are estimates of the hazard rate of overall service times that were performed at one-second resolution. The line in Figure 2 is a smoothed estimate of the hazard rate.

1. **(Solved)** How would you estimate the average service times of the four types, given only their survival functions? Only their hazard rates?
2. **(Submit)** Order the four service types by their average service time (from the smallest to the largest one).
3. **(Submit)** How would you estimate the variance of the service times, given only their survival functions?
   Hint. Let $f(x),\ x \geq 0$, denote the density of service time X. Integrate $E[X^2] = \int_0^\infty x^2 f(x)dx$ by parts in order to derive an expression that includes the survival function $S(x),\ x \geq 0$. Moreover, notice that $\lim_{t\to\infty} t^2 S(t) = 0$ for distributions with the finite second moment.
4. **(Submit)** How can you explain the regular increasing patterns at the right-hand part of Figure 2?

Remark to Question 4. The procedure of hazard-rate estimation was the following:

- Let A_k = number of customers with the service time equal to k seconds.
- Let η_k = number of customers with the service time larger or equal to k seconds.
- Then the estimates of the hazard rate (points in Figure 2) were calculated by:

$$\widehat{h(k)} \ = \ A_k/\eta_k \, .$$

Figure 1: Survival functions of service time by types.

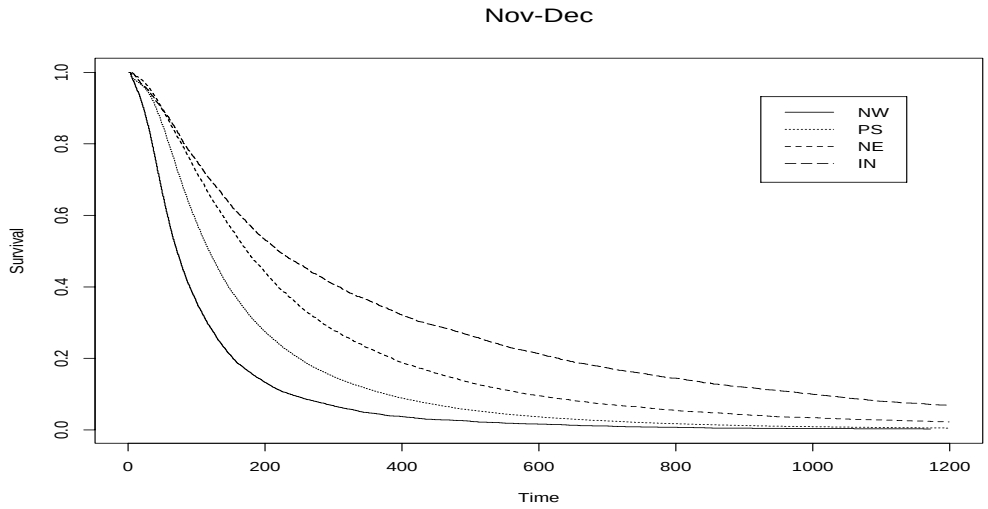
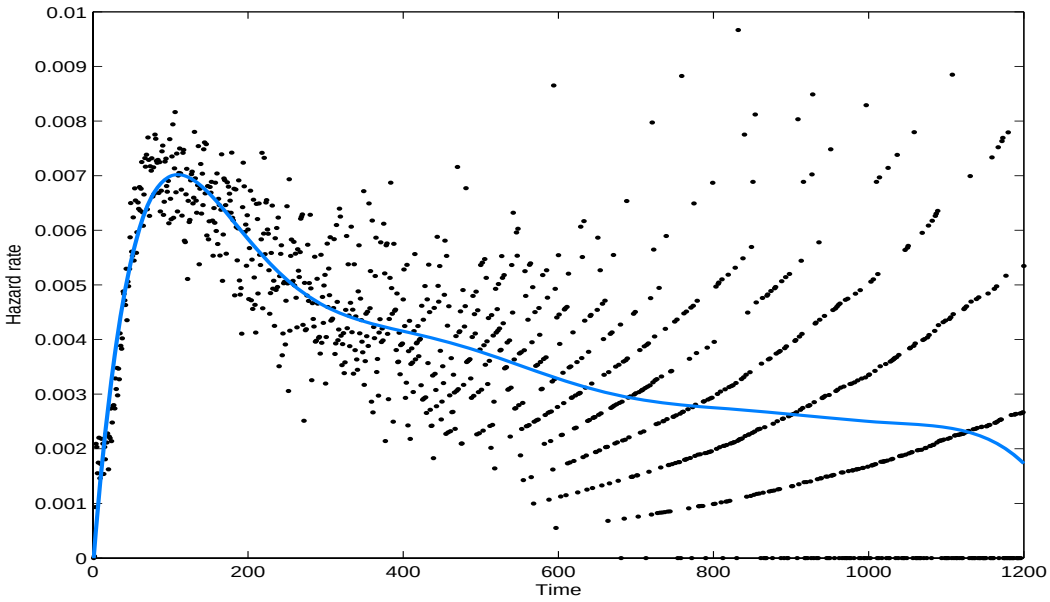


Figure 2: Hazard rate of service time.
Overall



Part 2. Empirical Analysis of Customers' Patience

Questions 1 and 2 are using Figures 1-8, presented below. The figures are based on **Call Center** Data (see Technical Appendix for details). They were obtained through the following procedure:

- The four major service types (PS, IN, NE and NW) were considered.
- Each figure is a scatterplot in which a point represents a time period of a fixed duration, covering the times between 7:00 and 24:00. For a given scatterplot, the duration is either 15 minutes or 30 minutes, the latter being used when the number of observations per 15 minutes was small. The scatterplots were created as follows.
- For each service type and time interval the following characteristics were calculated:
  - Average waiting times (from having left the VRU till start of service or abandonment).
  - Average waiting times given positive wait.
  - Probabilities of abandonment.
  - Probabilities of abandonment given positive wait.

  The results are summarized in the file *Patience.xls*, which you can download from our Internet site. (See Technical Appendix for a description of the file.)
- Four graphs depicting probabilities of abandonment versus average waiting times for the major service types were plotted (Figures 1-4).
- Four similar graphs given positive wait were plotted (Figures 5-8). (That is, we average here only over customers who were actually delayed in queue.)

Question 1.(Submit) Compare the two types of graphs (Figures 1-4 and 5-8, respectively). Describe common and different features, for each service type and overall. Provide explanations for the differences, if you can. Pay special attention to the IN service type (Figures 1 and 5).

Question 2.(Submit) Consider the following four models of customer's patience.

- **Standard exponential model.**
  The patience time (the time a customer is willing to wait) is exponential with parameter θ.
- **Exponential model with balking.**
  A customer that gets a busy signal abandons immediately with probability p (practically, several seconds will elapse before disconnecting). Otherwise, patience time is exponential with parameter θ.
- **Two phases of patience.**
  Patience $= X + Y$. The random variable (or constant) X corresponds to the time when a customer "feels Ok" and does not abandon. The random variable Y is exponential with parameter θ (the customer becomes impatient during this phase).
- **Adaptive behavior.**
  The patience of customers varies during the day (for example due to "waiting expectations", or different "cost of wait" during specific hours, etc.)

For each service type, choose the model that approximates reality in the best way. (We do not claim that a one-to-one correspondence between the four models and the four types prevails.) Support your conclusions by calculations if possible. Otherwise, present intuitive considerations. Which graphs are more helpful when you solve this question: conditioned on $\{Wait > 0\}$ or using averages over all customers? Explain your answer.
In the solution to Question 2, you may use Regression tools that are provided by Excel. We have found the functions INTERCEPT and SLOPE, and the regression module of "Data Analysis" most useful.

Question 3.(Submit) The following table summarizes estimates of average patience time, offered wait and service times. (The average patience and offered wait are based on the Kaplan-Meyer estimator.)

Type	Patience	Offered wait	Service time
IN	528 sec	268 sec	408 sec
NE	678 sec	143 sec	274 sec
NW	491 sec	229 sec	115 sec
PS	597 sec	117 sec	178 sec

Consider the following two definitions for **Patience Index**:

- Patience Index is equal to the average time that a customer is willing to wait per second of expected offered waiting time. (Assume that customers have some sort of "statistical knowledge" concerning the offered wait. Argue for which service types this assumption is reasonable).
- Patience Index is equal to the average time that a customer is willing to wait per second of expected service time.

Calculate both patience indices for the four types. Sort the types according to "patient customers" versus "impatient customers".
Which definition of Patience Index is more appealing to you? Why?

Technical Appendix.

Remarks for Questions 1-2. The analysis of PS-customers is based on data from 3 months: January, February and March. The analysis of the other 3 types is based on the overall 1999 data.
The NE records were assigned to 34 homogeneous time intervals, each interval half an hour long. The records of the other service types were assigned to 68 homogeneous time intervals, each one 15 minutes long. (The NE scatterplots with 68 points were too "noisy".)
The records were assigned to time intervals according to VRU-arrival times.

Remark for Question 3. The estimates of averages are based on November and December data.

Description of the file Patience_2013S.xls
The file contains eight graphs (Figures 1-8) and four tables in the worksheets "IN", "NE", "NW", and "PS". Each table consists of the following columns:

Time Time interval. For example "9:00" refers to interval 8:45-9:00. The following values correspond to this time interval.

Num Number of customers.

E[Wait] Average waiting time of customers.

$W > 0$ Number of customers with positive waiting time.

$P\{W > 0\}$ Fraction of customers that have been delayed.

$E(W|W > 0)$ Average waiting time of customers that have been delayed.

HANG Number of customers that abandoned.

$P\{Ab\}$ Fraction of customers that abandoned.

$P\{Ab|W > 0\}$ Fraction of customers that abandoned among customers with positive wait.

The last line of the table contains overall (daily) averages of the characteristics above.

Probability of Abandonment versus Average Waiting Time

Figure 1: **IN customers**

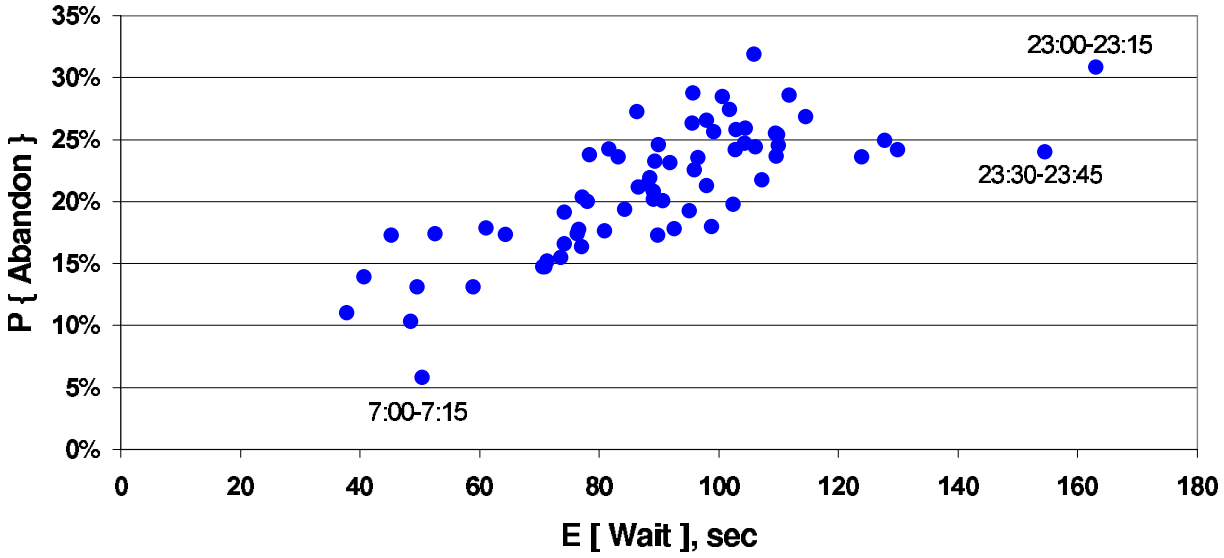


Figure 2: **NE customers**

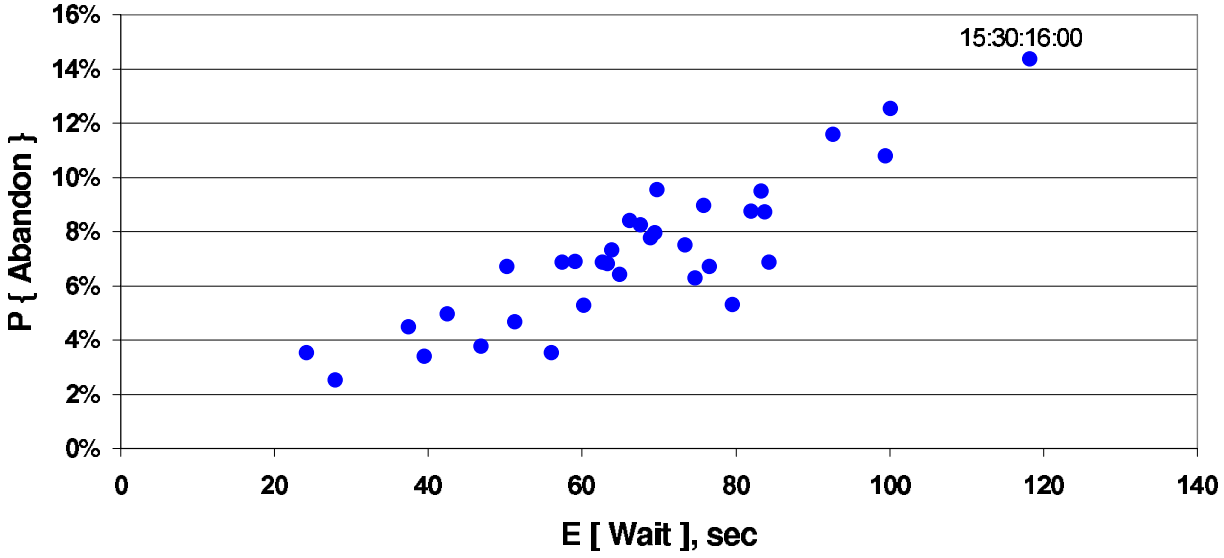


Figure 3: **NW customers**

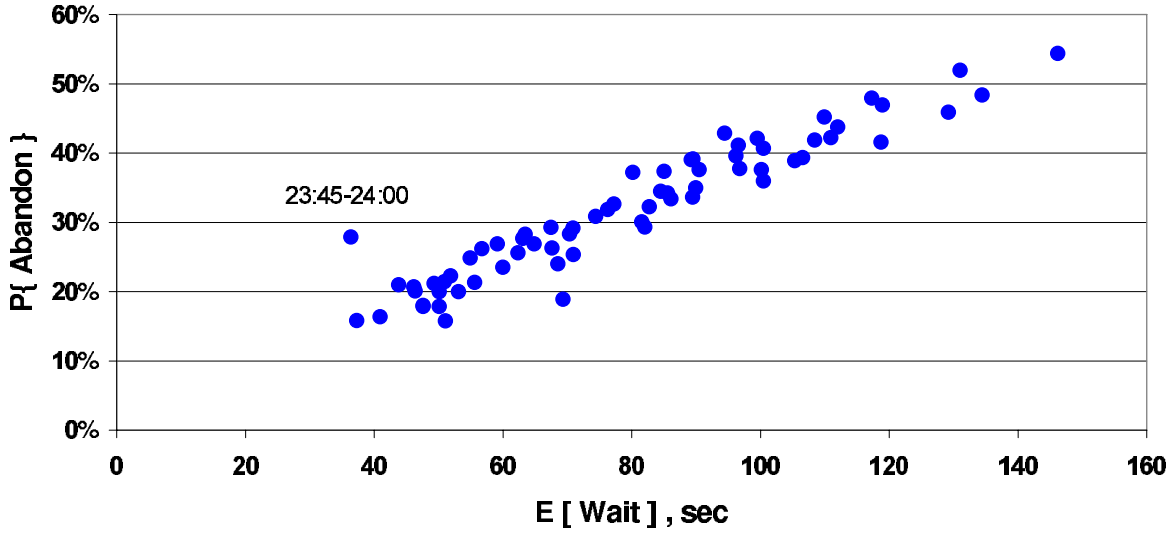
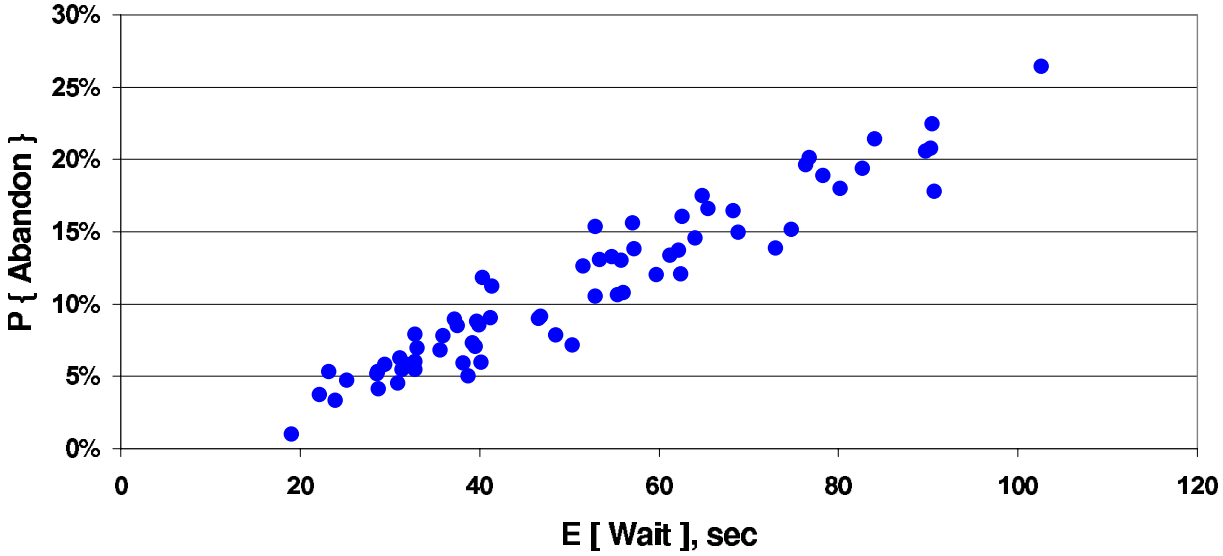


Figure 4: **PS customers**



Probability of Abandonment versus Average Waiting Time: Customers with Positive Waiting Time

Figure 5: **IN customers**

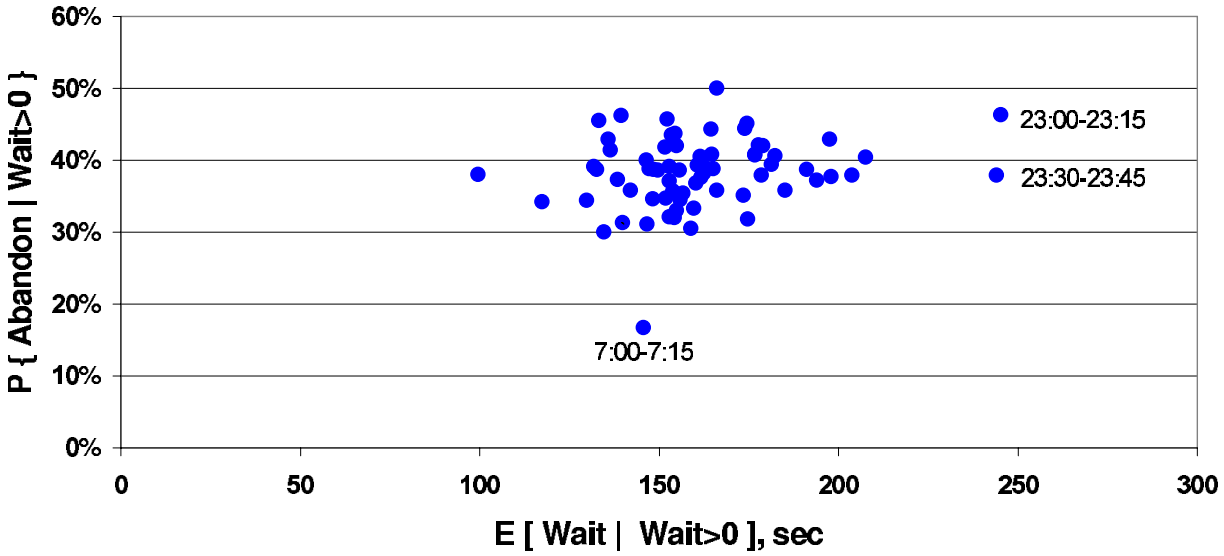


Figure 6: **NE customers**

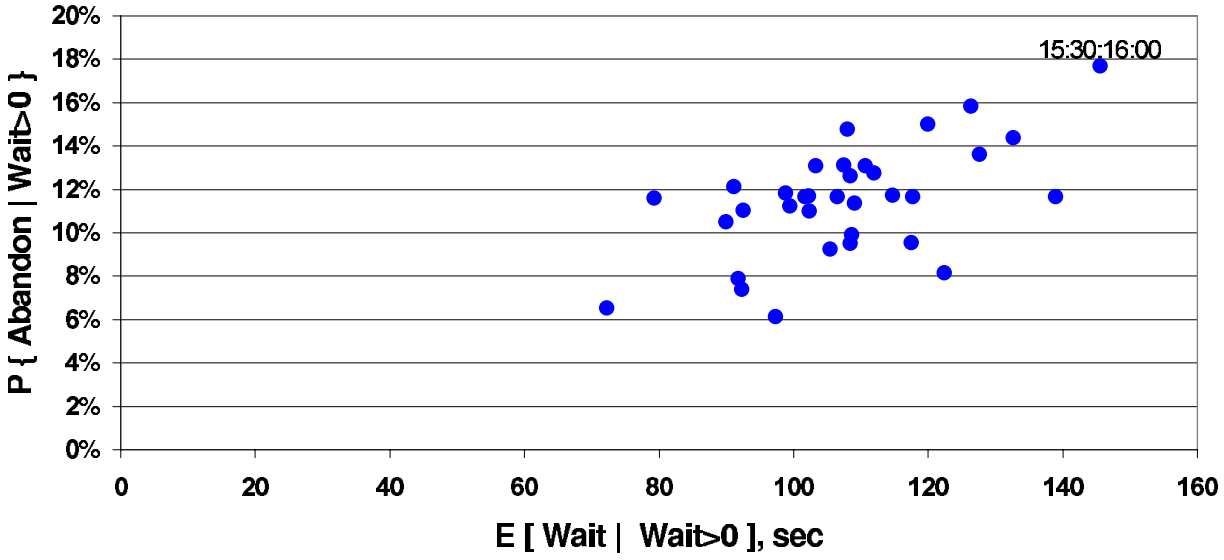


Figure 7: **NW customers**

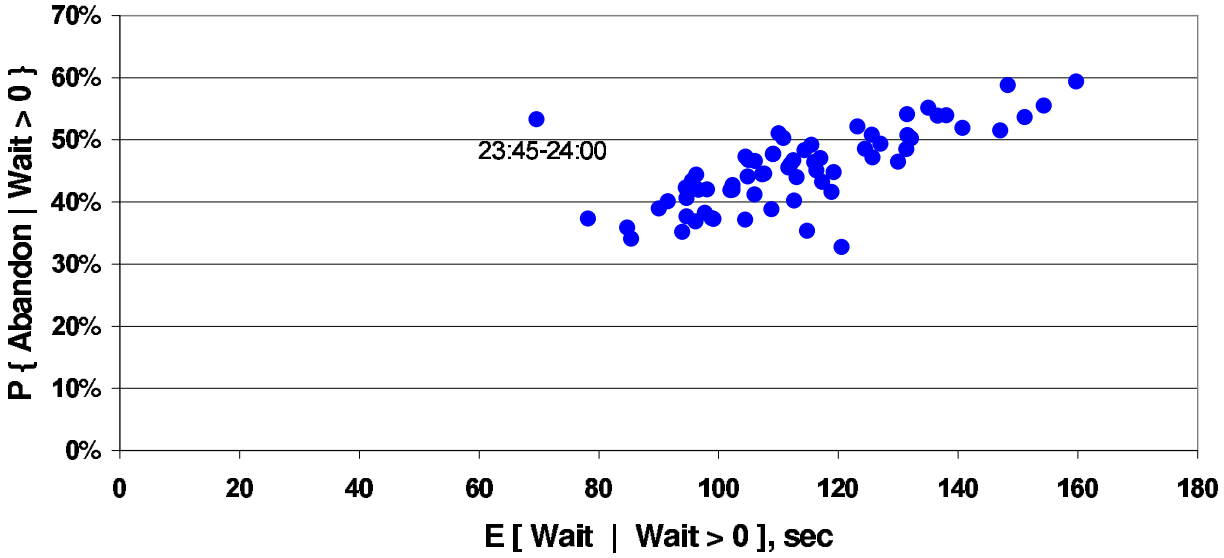


Figure 8: **PS customers**