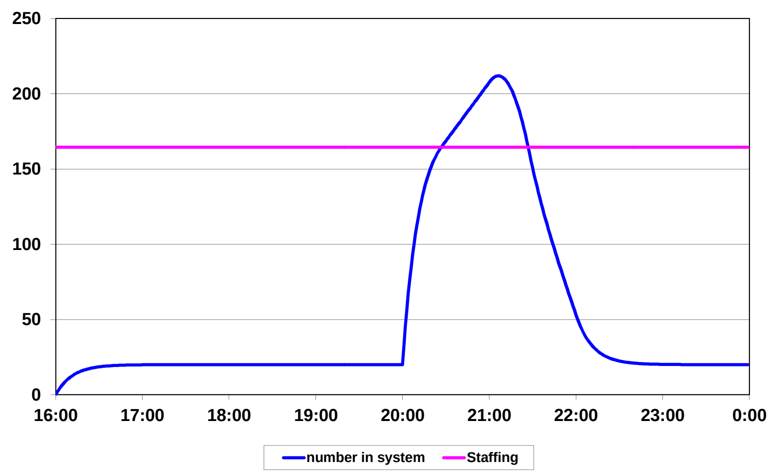
Homework 5 - Fluid Models. Partial Solution.

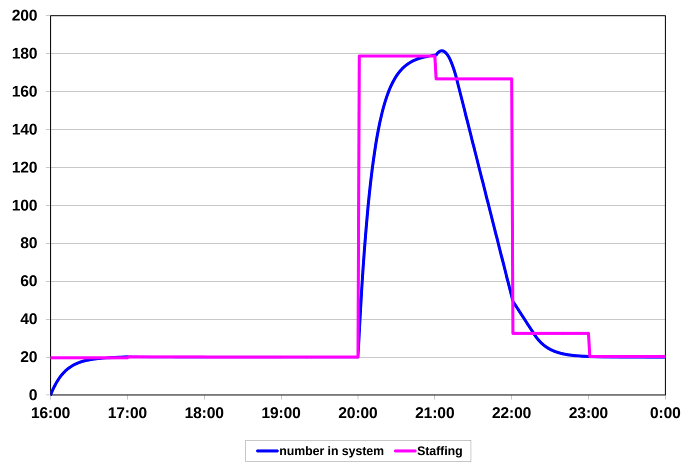
Solution of Question 1:



Here and below we use one-minute resolution in the difference equations. Staffing cost per minute $C = 0.7$.
Optimal solution: $N^* \approx 164$, Cost=53,799.

Solution of Question 2:

$N^* \approx (20, 20, 20, 20, 178, 166, 32, 20)$, Cost=19,139.
Note that the shift staffing enables a drastic cost decrease. The optimal staffing increases with the arrival peak at 8pm, remains almost at the same level between 8 and 10pm, and decreases significantly at 10pm. Note that during five hours the staffing remains at the equilibrium level $N = 20$, with the arrival rate equal to the total service rate.

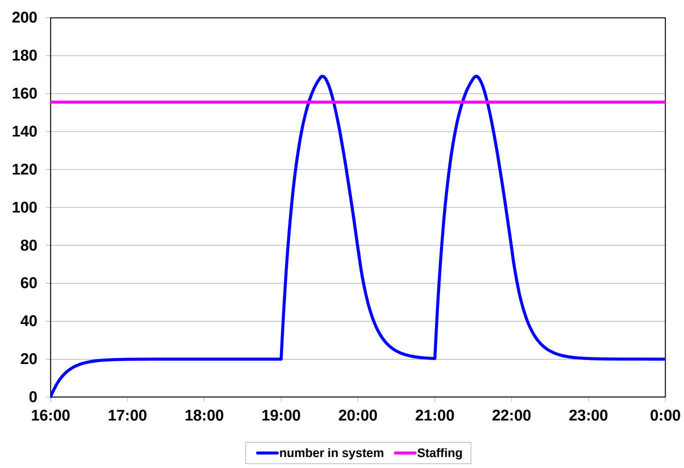


Solution of Question 7:

For $r = 80$ optimal staffing $N^* = 155$, Profit=108,308.
For $r = 150$ optimal staffing $N^* = 165$, Profit=246,917.

We observe that the optimal profit is higher than in the case of the one long arrival peak. It also turns out that less servers are needed in order to deal with a short arrival peak.

Reward-per-call $r = 80$



Partial Solution of Question 9:

Consider the old arrival rate.

$$\text{Prob}(abandon) = \frac{\text{total abandoned}}{\text{total arrived}} = \frac{\int_0^T \theta(Q_t - N_t)^+ dt}{\int_0^T \lambda_t dt} \approx ?$$

Partial Solution of Question 11:

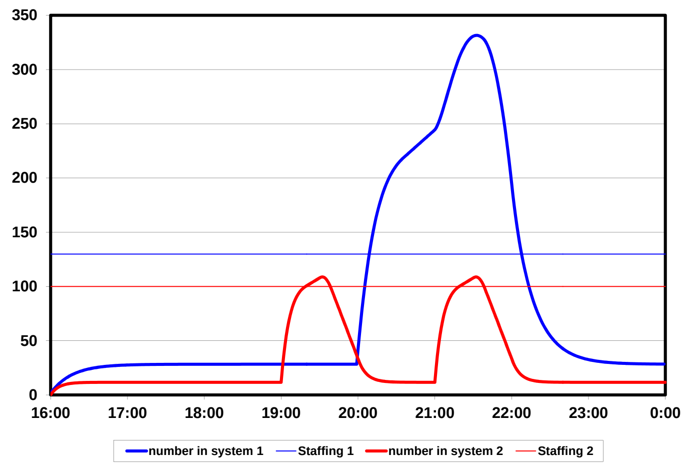
First policy:

$$\frac{dQ_1}{dt} = \lambda_1 - \mu_{11}\Big(Q_1 \wedge N_1\Big) - \mu_{12}\Big[(Q_1 - N_1)^+ \wedge (N_2 - Q_2)^+\Big]$$

$$\frac{dQ_2}{dt} = \lambda_2 - \mu_{22}\Big(Q_2 \wedge N_2\Big).$$

Partial Solution of Question 12:

First Policy: Regular customers are routed to station 1



For the first policy we get $N_1^* = 130$, $N_2^* = 100$ and overall cost is 86,183.