Service Engineering

Homework No. 2: Capacity Analysis. Little's Law.

Partial Solution.

Question 3. Since the scientist counts both "going in" and "going out", the average arrival rate

$$\lambda \;=\; \frac{0.061}{2} \text{ insects/hour} \;=\; 0.0305 \;.$$

The average number of insects in the cube $L = 0.0082$. Hence, the average duration of an insect visit

$$W \;=\; \frac{L}{\lambda} \;=\; 0.269 \text{ hours} \;=\; 967.8 \text{ seconds.}$$

Question 4. Arrival rate = total number of customers served. Therefore,

$$\lambda_{90} = 1.12 \cdot \lambda_{89} \qquad \text{and} \qquad L_{90} = 1.16 \cdot L_{89} \;.$$

Then

$$\frac{W_{90}}{W_{89}} \;=\; \frac{L_{90}}{\lambda_{90}} \cdot \frac{\lambda_{89}}{L_{89}} \;=\; \frac{1.16}{1.12} \;=\; 103.57\% \;.$$

So the average duration of a customer visit increased by 3.57% in 1990.

Question 5.
5.1 The average waiting time

$$W \;=\; \frac{L}{\lambda} \;=\; \frac{L_1 + L_2}{\lambda} \;=\; \frac{\lambda_1 W_1 + \lambda_2 W_2}{\lambda} \;=\; p_1 W_1 + p_2 W_2$$

is equal to the arithmetic weighted average.

5.2 In this case,

$$W \;=\; \frac{L}{\lambda} \;=\; \frac{L}{\lambda_1 + \lambda_2} \;=\; \frac{L}{L_1/W_1 + L_2/W_2} \;=\; \frac{1}{L_1/(L \cdot W_1) + L_2/(L \cdot W_2)}$$

$$=\; \frac{1}{p_1/W_1 + p_2/W_2}$$

is equal to the harmonic weighted average.

Question 6. There will be employed approximately

$$L \;=\; \lambda \cdot W \;=\; 100 \cdot 20 \;=\; 2000 \text{ accountants.}$$

Assumptions. λ and L will be constant till 2050. The system will reach steady state.

Question 7.

7.1 Estimate the average number of back-logged cases and the average number of cases that the judge resolves per month by

$$\bar{L} \;=\; \frac{\sum_{i=1}^{12} L_i}{12} \qquad \text{and} \qquad \bar{\lambda} \;=\; \frac{\sum_{i=1}^{12} \lambda_i}{12} \;,$$

respectively.
Then the average sojourn time of a case in the court per judge is (in months)

$$W \;=\; \frac{\bar{L}}{\bar{\lambda}} \;=\; \frac{\sum_{i=1}^{12} L_i}{\sum_{i=1}^{12} \lambda_i} \;.$$

7.2 Let
i = index of month;
j = index of judge.
The arrival rate to judge j is equal to

$$\lambda_j \;=\; \frac{1}{12}\lambda_{ij} \;.$$

The overall arrival rate to the system:

$$\lambda = \sum_j \lambda_j \;.$$

According to **7.1**, we can estimate the average sojourn time at judge j by

$$W_j \;=\; \frac{\sum_{i=1}^{12} L_{ij}}{\sum_{i=1}^{12} \lambda_{ij}} \;.$$

Now, generalizing **5.1**, we get

$$W \;=\; \sum_j \frac{\lambda_j}{\lambda} W_j \;.$$

Alternative way.

$$W \;=\; \frac{\sum_j \sum_{i=1}^{12} L_{ij}/12}{\sum_j \sum_{i=1}^{12} \lambda_{ij}/12} \;=\; \frac{\sum_j L_j}{\lambda} \;=\; \frac{\sum_j \lambda_j W_j}{\lambda} \;.$$

Question 9. The ER (emergency room) can be divided into 4 subsystems:

1. Queue for registration.
2. Registration.
3. Queue for doctors.
4. Doctors.

We know that the arrival rate λ to all subsystems is equal to 50 per hour $= 5/6$ per minute. Denote by W_i and L_i the waiting time and average number of customers in subsystem i. We know:

$$L_1 = 30;\ \ W_2 = 2 \text{ min};\ \ L_3 = 40;\ \ W_4 = 5 \cdot 90\% + 30 \cdot 10\% = 7.5 \text{ min}.$$

Now using Little formula for every subsystem we get

$$W_1 = \frac{30}{5/6} = 36 \text{ min};\ \ L_2 = \frac{5}{6} \cdot 2 = \frac{10}{6};\ \ W_3 = \frac{40}{5/6} = 48 \text{ min};\ \ L_4 = \frac{5}{6} \cdot 7.5 = 6.25.$$

If we denote by L and W the average number of customers and the average waiting time in ER respectively, then

$$L = L_1 + L_2 + L_3 + L_4 = 77.9;\ \ W = W_1 + W_2 + W_3 + W_4 = 93.5 \text{ min}.$$

Answer. A patience stays 93.5 minutes in the ER, on average. On average, 6.25 patients are being examined by doctors. There are 77.9 patients in the ER, on average.

Remark. The ratio

$$\frac{\text{service time}}{\text{waiting time}} = \frac{2 + 7.5}{36 + 48} = 0.11$$

is very small.