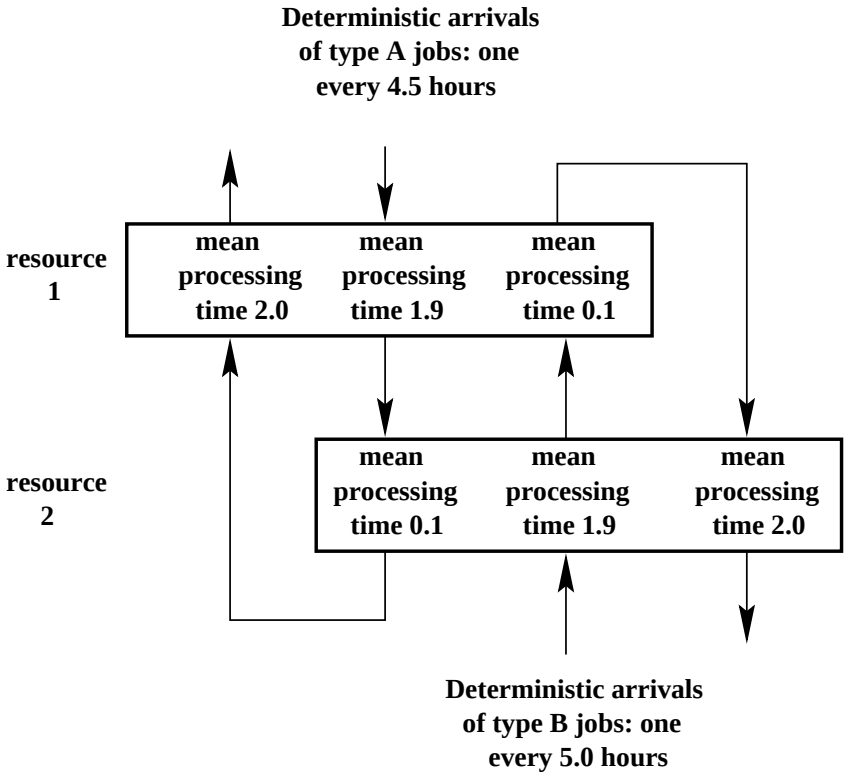
Homework No. 2: Capacity Analysis. Little's Law.

Submit questions: 1,2,8,10 and 11.

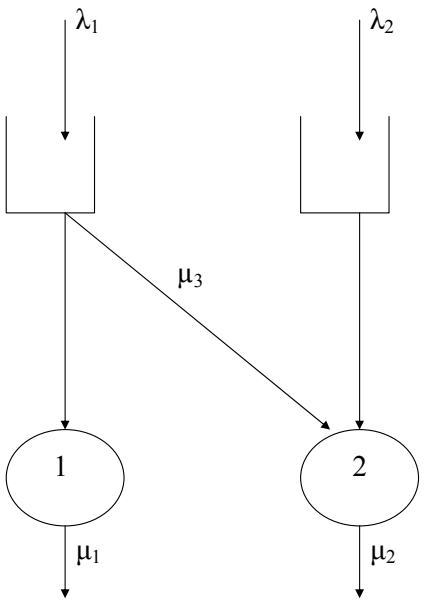
1. Consider an operation that processes two types of jobs, called type A and type B, and has only two processing resources—you may think of these as operators. The input rates, routing, and average processing times for the two job types are shown in the figure below.



(a) What is the average utilization of the two processing resources?

(b) Assume that type B jobs arrived at a rate of one every 3.5 hours. What is the average utilization of resource 1 and resource 2 in this case?

2. Stability



The figure above describes a model known as the *N-Model*. There are two types of customers, 1 and 2, with average arrival rates 1.138 and 0.32 customers per minute respectively. There are two servers for which the average service rates are: 1 customer per minute for server number one, 1 customer per minute for server number 2, when serving type 2 customers, and 0.5 customers per minute when serving type 1 customers.

The following scenario could be modeled by the above:
Think of server 2 as a multi-skilled server, capable of serving both types of customers, whereas server 1 specializes in service of type 1 customers, serving them more efficiently than server 2.

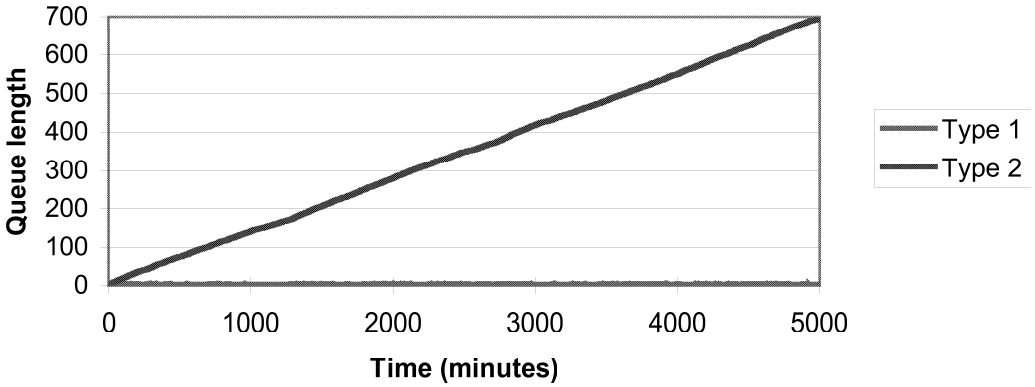
Answer the following questions:

(a) What is the least amount of help that **server 1 has to give to server 2** in order to ensure stability of the system ? i.e. what fraction (%) of the type 1 customers must be served by server number 1 ?

(b) Suggest a policy that achieves this goal, i.e. a policy that would guarantee that server 1 gives the necessary help to server 2.

(c) Suppose that type 2 customers are VIP's and type 1 are regulars. What would be a reasonable policy for operating the above system ?

Remark: (c) is much harder to answer when customers of type 1 are the VIP's. To see that, consider the following static priority policy: Server 2 serves type 1 customers as long as there are type 1 customers waiting and server 1 is busy. It can be seen in the graph below that with the same system parameters, and under the mentioned reasonable policy, the queue of type 2 would explode:



Littles's Law: $L = \lambda W$

3. A mad scientist has been studying the passage of insects through a certain cubic meter of air in central Minnesota, using automated instruments to continuously monitor insect positions. Her measurements show that, during the calendar year 1990, insects crossed the boundary of the "invisible cube" at an overall rate of 0.061 per hour, **either going in or going out**, and that the average number of insects in the cube was 0.0082. What was the average duration of an insect visit to the cube during 1990?

4. Goldie's Restaurant remains open 24 hours per day, 365 days per year. The total number of customers served in the restaurant during 1990 was 12% greater than the total for 1989. In each year, the number of customers in the restaurant was recorded at a large number of randomly selected times, and the average of those numbers in 1990 was 16% greater than the average in 1989. By how much did the average duration of a customer visit to the restaurant increase or decrease?

5. Consider a service system providing service to two types of customers. Type I customers spend, on average, W_1 units of time in the system and Type II customers spend an average time W_2. One is interested in calculating the average waiting time W of the whole population in the following two scenarios:

5.1 We are given (either by experience or through measurements) the quantities p_1, p_2 where $p_1 = \frac{\lambda_1}{\lambda}$, $p_2 = \frac{\lambda_2}{\lambda}$ are the proportions of customers of the two types within in-flow ($\lambda = \lambda_1 + \lambda_2,\ \lambda_i =$ in-flow rate of type i). Express W in terms of p_i and W_i.

5.2 The quantities p_1, p_2 are given, where $p_1 = \frac{L_1}{L}$, $p_2 = \frac{L_2}{L}$ are proportions in the system population. ($L = L_1 + L_2,\ L_i =$ average number in system of type i.) Express W in terms of p_i and W_i.

6. It is known that 100 candidates on average pass the annual qualification exam for accountants in Israel. An accountant works for 20 years on average (until retirement or professional change). How many accountants will be employed in Israel in 2050? Briefly formulate the assumptions that are used in your solution.

7. Assume that K judges work in Haifa's Labor court of law. The following data was collected for every judge:

7.1 Number of pending cases that await decision of judge j $(1 \leq j \leq K)$ at the end of the month:
$L_{1,j},\ L_{2,j},\ L_{3,j}, \ldots, L_{12,j}$, where
$L_{i,j}$ = number of back-logged cases of judge j at the end of month i, $i = 1, \ldots, 12$.

7.2 Number of cases that judge j $(1 \leq j \leq K)$ resolved during the months:
$\lambda_{1,j},\ \lambda_{2,j},\ \lambda_{3,j}, \ldots, \lambda_{12,j}$, where
$\lambda_{i,j}$ = number of cases, resolved by judge j during month i, $i = 1, \ldots, 12$.

The head-judge would like to estimate the sojourn time of a case in the court, per judge and overall. How can he use the above data without additional measurements? Explain your method and outline restrictions. %

8. It was mentioned during lecture that many organizations collect only *counting data* ("How many events took place?") without *time data* ("How long were the durations of those events?"). We illustrate the importance of collecting time data by the article that describes the treatment of cases in the Israel Military Court, enclosed of the following page.

8.1 The objective of the research described in the article is strictly operational: developing a model that will reflect, as good as possible, the workload on judges in Military Court. The judges in Military Court handle several types of cases: disorderly conduct (HAFAROT SEDER) cases , criminal (PLILIM) cases, traffic (TA'AVORA) cases and more. The research conductors claim that a judge workload can not be quantified based only on the amount of files he handles. Do you agree with this claim? Justify your answer.

8.2 Handling of the case can be divided into 3 consecutive phases:

- Pre hearing study and preparation.
- Hearing - Hearing the parties' pleadings and evidences and managing the deliberations in the court.
- Writing - Writing the judge's decision in the system.

We shall concentrate in the Hearing phase. The following table displays the average hearing time and standard deviation of diverse cases types.
Remark: the term "panel cases" (TIKEY HERKEV) in the table means the case was heard by three judges. Except for these cases, all other hearings are held by a sole judge. In 2010, 1000 disorderly conduct cases and 500 panel cases were opened in a specific Court. In 2011, at the same Court, 1500 disorderly conduct cases and only 400 panel cases were opened. What will be the increase/decrease of work load arising from managing the Court hearings? Write your calculations.

מחקר של שני שופטים בדק את משך הזמן הממוצע של דיונים בתיקים

אורך הדיונים בבימ"ש צבאי: 43 דקות בממוצע

חיים לוינסון

זמן שמיעה ממוצע של תיק בבית המשפט הצבאי במחנה עופר עומד על 43 דקות – כך עולה ממחקר שנערך בבתי המשפט.

מדובר בפרק הזמן נטו שבו מתנהל תיק בבית המשפט, לא כולל תקופות המתנה בין דיון לדיון. התיקים שדיוניהם נמשכים פרק זמן מרבי הם תיקים של נאשמים בפעילות הבלנית עוינת (פח"ע) בהרכב מורחב של שלושה שופטים.

המחקר נערך על ידי השופטים סגן־אלוף צבי לקה, חבר בית המשפט הצבאי לערעורים, ורב־סרן אמיר דהאן, שופט בית המשפט הצבאי יהודה והאחראי על הסטטיסטיקות בבתי המשפט.

השניים ערכו את המחקר כבסיס למאמרם "הצעה למודל מתמטי לקביעת מספר השופטים הרצוי עבור כמות התיקים המצויה", שפורסם בגיליון האחרון של כתב העת "הפרקליט".

השניים ביקשו לערוך בחינה של פרק הזמן שבו מתנהל תיק בבתי המשפט, בהתאם למאפייניו, כדי לחשב כמה תיקים ניתן לתת לשופט.

במסגרת המחקר הם בחנו 468 תיקים שונים ובאמצעות המחשב בדקו כמה זמן התנהלו הדיונים בכל תיק על כל שלביו: הקראה, דיונים שונים, הוכחות, טיעונים לעונש, הכרעת דין, גזר דין וכו'. הממצאים אינם כוללים זמן שיפוטי כמו הכנה לדיון וכתיבת פסקי דין.

הממצאים של לקה ודהאן מלמדים שהטיפול בתיקים בבית המשפט הצבאי בעופר מתנהל במהירות שיא: 43.72 דקות בממוצע בסך הכל. בחלוקה למאפייני התיק, נמצא כי תיקי פח"ע שיושבים בהם שלושה שופטים מתנהלים 63 דקות. תיק פח"ע שבו יושב שופט אחד נמשך כ-60 דקות. תיקים פליליים מתנהלים ב-46 דקות, תיקי הפרות סדר ב-29 דקות, תיקי תעבורה ב-18 דקות ותיקי שב"ח ב-45 דקות. השופטים בחנו גם את משך זמן הטיפול של תיקי מעצרים, העומד על 18 דקות. זהו נתון שיש לו מקבילה בישראל, שבו הטיפול בתיק מעצר עומד על שש דקות בלבד.

המחקר בחן כמה זמן התנהלו הדיונים בכל תיק על כל שלביו – מהקראה ועד גזר הדין

ממצאי המחקר משקפים את המציאות בבתי המשפט הצבאיים, שם הרוב המוחלט של התיקים לא מתנהלים כלל בבתי המשפט אלא מוכרעים בעסקת טיעון או בהודאה.

במאמר אחר שפרסמו בעבר, ברקו לקה ודהאן את נתוני שנת 2008 בבתי המשפט הצבאיים והתגלה כי 95.5% מ-4,000 התיקים שנירונו באותה השנה הסתיימו בהסדר או בהודאה, ללא התערבות כלל של בית המשפט הצבאי.

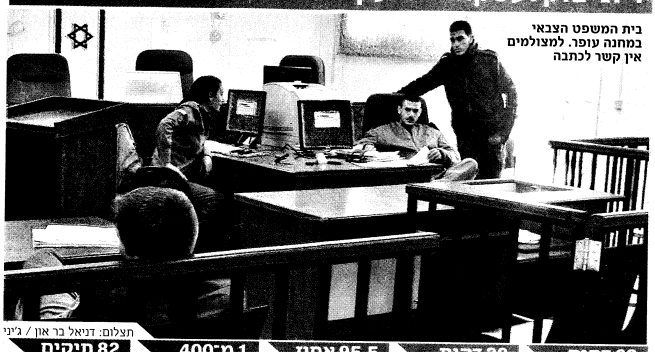
כפי שפורסם באחרונה ב"הארץ", דו"ח בתי המשפט הצבאיים בישראל לשנת 2010 לימד כי רק 0.26% מהתיקים מסתיימים בזיכוי מוחלט של הנאשם. עם זאת, בבתי המשפט טוענים כי מדובר בנתון מטעה בשל ריבוי עסקאות הטיעון.

גורם במערכת אמר ל"הארץ" כי בניתוח פרטיני האישום שנשארו במחלוקת בין הצדדים, שיעור הזיכוי עומד על 29%. כך או כך, בשנת 2010, רק ב-82 תיקים נוהלו הוכחות ונכתבה הכרעת דין מפורטת.

עורך דין המרבה להופיע בבתי המשפט הצבאיים אמר ל"הארץ" כי "הנתונים מסתדרים. ידוע שמעטים מבקשים לנהל תיק על הפרות סדר. זמן ניהול המשפט, שבמהלכו יושב הנאשם במעצר עד תום ההליכים, ארוך יותר מפרק הזמן שניתן לקבל בעסקת טיעון. לכן תיק הפרות סדר מתנהל בפחות מחצי שעה. אין טעם לנהל אותו".

דובר צה"ל סירב לאשר שיחה עם עורך המחקר, רב־סרן דהאן, בנימוק ששופטים אינם נוהגים לשוחח עם עיתונאים מטעמים אתיים.

דיוני בזק ועסקאות טיעון



בית המשפט הצבאי במחנה עופר. למצולמים אין קשר לכתבה

תצלום: דניאל בר און / גיני

63 דקות	29 דקות	95.5 אחוז	1 מ־400	82 תיקים
משך דיון ממוצע בתיק פח"ע בהרכב של שלושה שופטים	משך דיון ממוצע בתיק העוסק בפלסטיני שהפר את הסדר	שיעור התיקים שהסתיימו ב-2008 בהסדר טיעון או הודאה	מספר התיקים ב-2010 שהסתיימו בזיכוי הנאשם	מספר המקרים ב-2010 שבהם התקבלה הכרעת דין

Case Type	Average Hearing Time (minutes)	Standard Deviation
Panel Cases (TIKEY HERKEV)	189.2 (This result should be multiplied by the number of judges in the panel).	175.32
Destructive Activity (PACHA)	60.12	47.94
Criminal Cases (PLILIM)	46.096	43.7
Disorderly Conduct (HAFAROT SEDER)	29.34	24.55
Traffic (TA'AVORA)	18.8	6.7
Illegal Immigrants (SHABACH)	45.31	13.37
Custody Cases (MA'ATZARIM)	18.17	13.19

8.3 During January 2012, 30 hostile destructive activity (PACHA) cases were opened and 25 such cases were closed in a specific court. In the end of the month there were 75 such cases currently opened in the same court. Out of the total time a case is opened, what is the percentage of time that is dedicated for hearing the case?

Remark: the duration, in hours, in which a case is considered to be in court, will be calculated only by standard working hours in a month - 20 working days, 8 working hours a day, a total of 160 working hours a month

Problems 9-11 are adapted from *Managing Business Process Flows*, by R.Anupindi, S.Chopra, S.Deshmukh, J.Van Mieghem, E.Zemel. Prentice Hall, 1999.

9. A hospital emergency room (ER) is currently organized so that all patients register through an initial check-in process. At his or her turn, each patient is seen by a doctor and then exits the process, either with a prescription or with admission to the hospital. Currently, 50 people per hour arrive at the ER, 10% of whom are admitted to the hospital. On average, 30 people are waiting to be registered and 40 are registered and waiting to see a doctor. The registration process takes, on average, 2 minutes per patient. Among patients who receive prescriptions, average time spent with a doctor is 5 minutes. Among those admitted to the hospital, average time is 30 minutes. On average, how long does a patient stay in the ER? On average, how many patients are being examined by doctors? On average, how many patients are in the ER?

10. A triage system has been proposed for the ER described in Problem 9. As mentioned, 50 patients per hour arrive at the ER. Under the proposed triage plan, patients who are entering will be registered as before. They will then be quickly examined by a nurse practitioner who will classify them as Simple Prescriptions or Potential Admits. While Simple Prescriptions will move on to an area staffed for regular care, Potential Admits will be taken to the emergency area. Planners anticipate that initial examination by a triage nurse will take 3 minutes. They expect that, on average, 20 patients will be waiting to register and 5 will be waiting to be seen by a triage nurse. Recall that registration takes an average of 2 minutes per patient. Planners expect the Simple Prescription area to have, on average, 15 patients waiting to be seen. As before, once a patient's turn comes, each will take 5 minutes of a doctor's time. The hospital anticipates that, on average, the emergency area will have only 1 patient waiting to be seen. As before, once that patient's turn comes, he or she will take 30 minutes of a doctor's time. Assume that, as before, 90% of all patients are Simple Prescriptions. Assume, too, that triage nurses are 100% accurate in their classifications. Under the proposed plan, how long, on average, will a patient stay in the ER? On average, how long will a Potential Admit stay in the ER? On average, how many patients will be in the ER? What is the least number of nurses needed to accommodate the load?

11. Refer again to Problems 9 and 10. Once the triage system is put in place, it performs quite close to expectations. All data conform to planners' expectations except for one set—the classifications made by the nurse practitioner. Assume that the triage nurse has been sending 91% of all patients to the Simple Prescription area when in fact only 90% should have been so classified. The remaining 1% are discovered when transferred to the emergency area by a doctor. As a result, 19 patients, on average, are waiting in the Simple Prescription area (versus 15 patients in Problem 11). Assume that wrongly classified patients spend, on average, 10 minutes at their first doctor (Simple Prescriptions area) and 30 minutes at their second doctor in the emergency area. In Addition, notice that the wrongly classified patients visit two queues at the last stage. Assume all other information from Problem 11 is valid. On average, how long does a patient stay in the ER? On average, how long does a Potential Admit stay in the ER? On average, how many patients are in the ER?