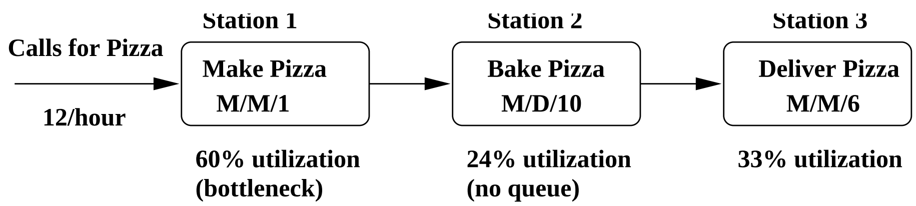
Service Engineering

Homework 10
Partial Solution

Part 1. "Anonymous Pizza" Case Study.

Question 1. The process flow diagram:



We represent the service process in the form of a simple queueing network with three stations. Making pizza is the bottleneck operation (note also that only one server is working there which adds to stochastic variability).
According to Burke theorem the departures from Station 1 are exponential. Since Station 2 provides the constant-time service that starts immediately, departures from Station 2 are exponential too.

Assumptions

- Bake time is a constant 12 minutes.
- Pizza is placed immediately in oven after making (no queue at a second station due to low utilization).
- Pizzas are delivered one at a time.
- Delivery time is assumed to be exponential with 5 minutes mean.
- Sojourn times at Station 1 and Station 3 are independent.

Remark. Note the difference between the *delivery time* (exponential with 5 minutes mean) and the *service time at station 3* (time until driver's return: exponential with 10 minutes mean). In fact, we assume that the delivery time is equal to the return time.

Part 2. M/G/n Queues. Queueing performance for varying service-time distributions.

2.1 Let X denote the service time. Its expected value is equal to 2/3 hour. The missing parameters are:

a. $X \sim \exp(\mu)$; $\mu = \frac{1}{\mathrm{E}X} = 1.5.$

b. $X \sim \text{Pareto}(\alpha = 2.5, k)$; $k = (\alpha - 1)\mathrm{E}X = 1.$

c. $X \sim \text{Pareto}(\alpha = 1.5, k)$; $k = (\alpha - 1)\mathrm{E}X = \frac{1}{3}.$

d. $X \sim \text{Lognormal}(\mu, \sigma = 1)$; $\mathrm{E}X = e^{\mu + \sigma^2/2}.$

Therefore, $\mu = \ln(2/3) - 0.5 = -0.905.$